Stabilizing switching control of power converters: the lossy line and nonlinear case

Marius Zainea†, Arjan van der Schaft‡, Jean Buisson†
†Supelec-IETR Rennes, Hybrid Systems Control Group, France
‡Dept. of Mathematics, University of Groningen, The Netherlands

1. Port-Hamiltonian representation of power-converters
2. Controlled equilibria and Lyapunov functions
3. The transmission line as a port-Hamiltonian system
4. Power converter + transmission line + load

This work was partially supported by HYCON.
Port-Hamiltonian representation of power converters

Example: Boost converter

Figure 1: Boost circuit with clamping diode

The circuit consists of a capacitor $C$ with electric charge $q_C$, an inductor $L$ with magnetic flux linkage $\phi_L$, and a resistive load $R$, together with an ideal diode and an ideal switch $S$, with switch positions $s = 1$ (switch closed) and $s = 0$ (switch open).
The voltage-current characteristics of the ideal diode and switch are depicted in Figure 2.

\[
\begin{align*}
\nu_D & = 0, \quad \nu_D \leq 0, \quad i_D \geq 0.
\end{align*}
\]

Figure 2: Voltage-current characteristic of an ideal diode and ideal switch
This yields the port-Hamiltonian model (with
\( H(q_C, \phi_L) = \frac{1}{2C}q_C^2 + \frac{1}{2L}\phi_L^2 \)):

\[
\begin{bmatrix}
\dot{q}_C \\
\dot{\phi}_L
\end{bmatrix} = \begin{bmatrix}
-\frac{1}{R} & 1 - s \\
 s - 1 & 0
\end{bmatrix} \begin{bmatrix}
\frac{\partial H}{\partial q_C} = \frac{q_C}{C} \\
\frac{\partial H}{\partial \phi_L} = \frac{\phi_L}{L}
\end{bmatrix} + \begin{bmatrix}
0 \\
1
\end{bmatrix} E + \begin{bmatrix}
 s i_D \\
(s - 1)v_D
\end{bmatrix}
\]

\[
I = \frac{\phi_L}{L}
\]

By disregarding the 'discontinuous modes' (normal operation), that is, assuming that when the switch is closed \((s = 1)\) the diode is open \((i_D = 0)\), while if the switch is open \((s = 0)\) the diode is closed \((v_D = 0)\),
we obtain the **switching port-Hamiltonian system**

\[
\begin{bmatrix}
\dot{q}_C \\
\dot{\phi}_L
\end{bmatrix} = \begin{bmatrix}
-\frac{1}{R} & 1 - s \\
 s - 1 & 0
\end{bmatrix} \begin{bmatrix}
\frac{\partial H}{\partial q_C} = \frac{q_C}{C} \\
\frac{\partial H}{\partial \phi_L} = \frac{\phi_L}{L}
\end{bmatrix} + \begin{bmatrix}
0 \\
1
\end{bmatrix} E
\]

\[
I = \begin{bmatrix}
0 & 1
\end{bmatrix} \begin{bmatrix}
\frac{\partial H}{\partial q_C} = \frac{q_C}{C} \\
\frac{\partial H}{\partial \phi_L} = \frac{\phi_L}{L}
\end{bmatrix} = \frac{\phi_L}{L}
\]
In general, a switching port-Hamiltonian system without algebraic equality and inequality constraints is defined as

\[
\dot{x} = F(\rho)z + g(\rho)u, \quad z = \frac{\partial H}{\partial x}(x)
\]

\[
y = g^T(\rho)z
\]

with

\[
F(\rho) = J(\rho) - R(\rho), \quad J(\rho) = -J^T(\rho), \quad R(\rho) = R^T(\rho) \geq 0
\]

Note that the system is passive for every switching sequence:

\[
\frac{d}{dt}H = -\frac{\partial^T H}{\partial x}(x)R(\rho)\frac{\partial H}{\partial x}(x) + u^T y \leq u^T y
\]
Basic property of power converters in normal operation

Consider the port-Hamiltonian representation

\[ \dot{x} = F(\rho)z + g(\rho)E + g_l u, \quad z = \frac{\partial H_p}{\partial x}(x) \]
\[ y = g_l^T z \]

with vector of Boolean variables \( \rho \in \{0, 1\}^p \), \( H_p(x) \) the total stored electromagnetic energy, and \( (u, y) \) the current-voltage pair over the resistive load.
Let $x_0$ be an equilibrium of the averaged model, that is

$$F(\rho_0)z_0 + g(\rho_0)E + glu_0 = 0, \quad z_0 = \frac{\partial H_p}{\partial x}(x_0)$$

for some $\rho_0 \in [0, 1]^p$ and $u_0$. Then

$$\dot{x} = F(\rho)(z - z_0) + F(\rho)z_0 + g(\rho)E + glu$$

$$= F(\rho)(z - z_0) + [F(\rho) - F(\rho_0)]z_0 + [g(\rho) - g(\rho_0)]E + gl(u - u_0)$$

$$+ F(\rho_0)z_0 + g(\rho_0)E + glu_0$$

$$= F(\rho)(z - z_0) + [F(\rho) - F(\rho_0)]z_0 + [g(\rho) - g(\rho_0)]E + gl(u - u_0)$$
For many power converters in normal operation we know that

\[ F(\rho) - F(\rho_0) = \sum_{i=1}^{p} F_i(\rho_i - \rho_{0i}) \]
\[ g(\rho) - g(\rho_0) = \sum_{i=1}^{p} g_i(\rho_i - \rho_{0i}) \]

and thus

\[ \dot{x} = F(\rho)(z - z_0) + \sum_{i=1}^{p} [F_i z_0 + g_i E](\rho_i - \rho_{0i}) + g_l(u - u_0) \]

Take as Lyapunov/storage function

\[ V(x) := H_p(x) - (x - x_0)^T \frac{\partial H_p}{\partial x}(x_0) - H_p(x_0) \]

Then

\[ \frac{d}{dt} V(x) = [\frac{\partial H_p}{\partial x}(x) - \frac{\partial H_p}{\partial x}(x_0)]^T \dot{x} = (z - z_0)^T \dot{x} = (z - z_0)^T F(\rho)(z - z_0) \]
\[ + \sum_{i=1}^{p} (z - z_0)^T [F_i z_0 + g_i E](\rho_i - \rho_{0i}) + (z - z_0)^T g_l(u - u_0) \]

with \((z - z_0)^T F(\rho)(z - z_0) = -(z - z_0)^T R(\rho)(z - z_0) \leq 0.\]
Thus at any time we can choose the Boolean variables $\rho_i \in \{0, 1\}$ in such a manner that
\[
\frac{d}{dt} V(x) \leq (z - z_0)^T g_l(u - u_0)
\]
implying passivity of the switched system with respect to the input vector $u - u_0$ and output vector $y - y_0 = g_l^T(z - z_0)$.

As a consequence, if the converter is terminated on a resistive load $R$ (and hence the equilibrium $x_0$ should be such that $y_0 = -Ru_0$) then the switched converter is generally asymptotically stable around $x_0$. Thus the voltage over the resistive load can be stabilized around any set-point. (For the linear case, cf. Buisson, Cormerais, Richard.)

**Remark 1** Note that for linear capacitors and inductors we have
\[
H_p(x) = \frac{1}{2} x^T Q x, \quad V(x) = \frac{1}{2} (x - x_0)^T Q(x - x_0)
\]
Power converter connected to the load via transmission line

Figure 3: The Boost converter with a transmission line
Port-Hamiltonian representation of the transmission line

Let $s \in [0, 1]$ be the spatial variable. The energy variables are the charge density $Q = Q(t, s)\, ds$, and the flux density $\varphi = \varphi(t, s)\, ds$. The total energy stored in the transmission line is

$$H_l(Q, \varphi) = \int_0^1 \frac{1}{2} \left( \frac{Q^2(t, s)}{C_l} + \frac{\varphi^2(t, s)}{L_l} \right) ds$$

where $C_l$ and $L_l$ are the distributed capacitance and inductance of the line. The voltage and current at any position $s \in [0, 1]$ are

$$V(t, s) = \frac{Q(t, s)}{C_l}$$
$$I(t, s) = \frac{\varphi(t, s)}{L_l}$$
The lossy transmission line is given by the telegrapher’s equations
\[
\frac{\partial Q}{\partial t} = -\frac{\partial I}{\partial s} - G_l V(s, t)
\]
\[
\frac{\partial \varphi}{\partial t} = -\frac{\partial V}{\partial s} - R_l I(s, t),
\]
where \(G_l \) and \(R_l\) are the distributed conductance and the distributed resistance of the line. Since the transmission line is terminated on a resistive load \(R_L\)
\[
V(t, 1) = R_L I(t, 1)
\]
while \(V(t, 0)\) and \(I(t, 0)\) is the voltage, respectively the current, at the beginning of the line, as to be connected to the voltage\(=\)current pair \((y, u)\) of the power converter.

This is a port-Hamiltonian system satisfying
\[
\frac{d}{dt} H_l \leq -R_L V^2(t, 1) + uy
\]
Power converter + transmission line + load

The total system is a port-Hamiltonian system with Hamiltonian $H_p + H_l$.

An equilibrium $(V_0(s), I_0(s))$ for the line satisfies

$$\frac{\partial Q}{\partial t} = \frac{\partial \varphi}{\partial t} = 0,$$

and thus is a solution of

$$\begin{pmatrix}
\frac{\partial V_0}{\partial s}(s) \\
\frac{\partial I_0}{\partial s}(s)
\end{pmatrix} = \begin{pmatrix}
0 & -R_l \\
-G_l & 0
\end{pmatrix} \begin{pmatrix}
V_0(s) \\
I_0(s)
\end{pmatrix},$$

satisfying additionally the boundary condition

$$V_0(0) = y_0$$
$$V_0(0) = u_0$$
$$V_0(1) = R_L I_0(1)$$
An equilibrium of the system \textit{power converter + transmission line + load} is thus given by \((z_0, V_0, I_0)\) linked by certain \(u_0, y_0\) (the equilibrium current and voltage at the left-end of the line).

Take as Lyapunov function of the total system

\[
V(x, Q, \phi) := H(x) - (x - x_0)^T \frac{\partial H}{\partial x}(x_0) - H(x_0) + H_l(Q - Q_0, \phi - \phi_0)
\]

It follows that

\[
\frac{d}{dt} V = (z - z_0)^T F(\rho)(z - z_0) + \sum_{i=1}^{p} (z - z_0)^T [F_i z_0 + g_i E](\rho_i - \rho_{0i}) \\
-(V(t, 1) - V_0)^2 / R_L - \int_{0}^{1} \left[ (V - V_0)^2 G_l + (I - I_0)^2 R_l \right] ds
\]

and, as before, we can make the right-hand side negative by choice of the Boolean variables \(\rho_i \in \{0, 1\}\) (depending only on the values of the state variables of the power converter).
Conclusions

1. Stabilizing switching control strategy for linear power converters has been extended to
   - Lossy transmission line between power converter and resistive load.
   - Nonlinear capacitors and inductors.

2. How to extend to nonlinear resistive elements?

3. How to deal with discontinuous modes?