Iterative Bidding in Electricity Markets: Rationality and Robustness

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Joint work with: Jorge Cortés (UCSD)

Tertiary control/dispatch

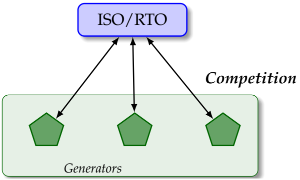
Current practice:

- gens. submit (closed) bids
- payment vs supply curves
- the ISO solves the following problem

Security constrained OPF

$$\underset{P}{\text{minimize}} \quad \text{payment}(P)$$
$$\text{subject to} \quad P \in \mathcal{F}$$

- ISO sends P_i to each gen. i
- this defines profit for each gen.



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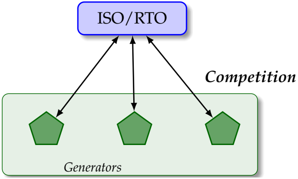

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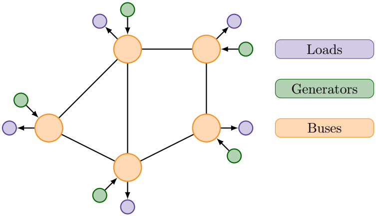
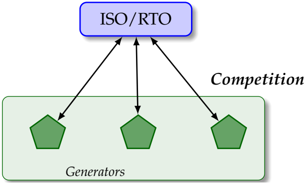
Question: What if generators are allowed to iterate bids before final dispatch?

- dynamic analysis (prescribe possible behavior)
- analyze its rationality and robustness
- interconnection with the physical dynamics

Problem statement: DC Optimal Power Flow





Setup

- N_b buses, N (generators)
- cost $f_n(x) = a_n x^2 + c_n x$, $a_n > 0$, $c_n \geq 0$
- load $y_i \geq 0$ at bus i

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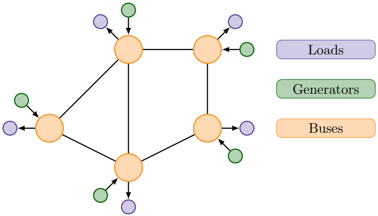

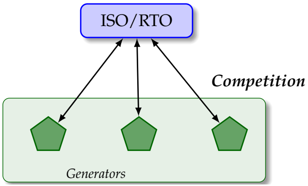

DC Optimal Power Flow

$$
\begin{aligned}
\underset{(x,z)}{\text{minimize}} \quad & \sum_{n=1}^{N} f_n(x_n) \\
\text{subject to} \quad & \sum_{j \in \mathcal{N}_i^+} z_{ij} - \sum_{j \in \mathcal{N}_i^-} z_{ij} = \sum_{n \in G_i} x_n - y_i, \ \forall i \\
& -\overline{z}_{ij} \leq z_{ij} \leq \overline{z}_{ij}, \ \forall (i,j) \\
& x \geq 0
\end{aligned}
$$

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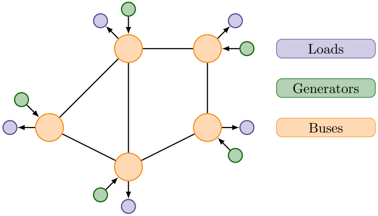

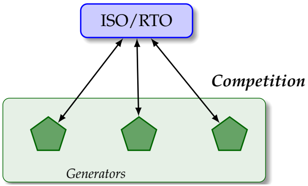

OPF

$$\begin{aligned} \underset{(x,z)}{\text{minimize}} \quad & \sum_{n=1}^{N} f_n(x_n) \\ \text{subject to} \quad & A \begin{bmatrix} x \\ z \end{bmatrix} = d \\ & G \begin{bmatrix} x \\ z \end{bmatrix} \leq h \end{aligned}$$

Strategic OPF

$$\begin{aligned} \underset{(x,z)}{\text{minimize}} \quad & \sum_{n=1}^{N} b_n x_n \\ \text{subject to} \quad & A \begin{bmatrix} x \\ z \end{bmatrix} = d \\ & G \begin{bmatrix} x \\ z \end{bmatrix} \leq h \end{aligned}$$

Electricity market game

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Electricity market game

1. Players: the set of generators $\{1, \ldots, N\}$
2. Action: for each player n, bid $b_n \in \mathbb{R}_{\geq 0}$
3. Utility: for each player n, payoff $u_n(b_n, x_n^{\text{opt}}(b)) = b_n x_n^{\text{opt}}(b) - f_n(x_n^{\text{opt}}(b))$

Electricity market game

OPF

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Beyond static analysis

1. how do generators reach Nash equilibria?
2. to what extent is the behavior rational?
3. how robust to disturbances, deviations, and collusion?

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Beyond static analysis

1. how do generators reach Nash equilibria?
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Is iterative bidding good?

Existence & uniqueness of efficient NE

b^* is a **Nash equilibrium** if $\exists$ optimizer $(x^{\text{opt}}(b^*), z^{\text{opt}}(b^*))$ of S-OPF

$$u_n(b_n, x_n^{\text{opt}}(b_n, b_{-n}^*)) \leq u_n(b_n^*, x_n^{\text{opt}}(b^*))$$

for all $n \in \{1, \ldots, N\}$, all $b_n \in \mathbb{R}_{\geq 0}$, and all optimizers $(x^{\text{opt}}(b_n, b_{-n}^*), z^{\text{opt}}(b_n, b_{-n}^*))$

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b^* is **efficient** if OPF optimizer (x^*, z^*) is S-OPF optimizer for b^*, and

$$x^*_n = \mathrm{argmax}_{x \geq 0} b^*_n x - f_n(x) \quad \text{for all } n \in \{1, \ldots, N\}$$

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Lemma (Existence of efficient Nash equilibrium)

If each bus has either more than one generator or none, then $\exists$ efficient Nash equilibrium

Existence & uniqueness of efficient NE

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$$x_n^* = \mathrm{argmax}_{x \geq 0} b_n^* x - f_n(x) \quad \text{for all } n \in \{1, \ldots, N\}$$

Lemma (Uniqueness of efficient Nash equilibrium)

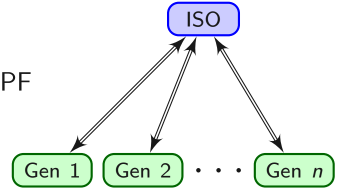
If optimizer x^* of OPF satisfies $x_n^* > 0$ for all n, and the existence condition is satisfied, then $\exists!$ efficient Nash equilibrium

Bid Adjustment Algorithm

Initialize: Each generator selects $b_n(1) \geq c_n$

At round k, ISO executes:

- Receive $b_n(k)$ from each n
- Find a solution $(x^{\text{opt}}(k), z^{\text{opt}}(k))$ to the S-OPF
- Send $x_n^{\text{opt}}(k)$ to each n



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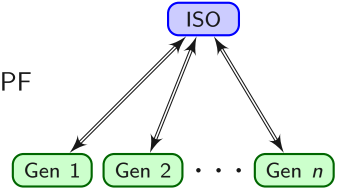

At round $k+1$, generator n executes:

- Receive $x_n^{\mathrm{opt}}(k)$ from ISO
- Set $b_n(k+1) = [b_n(k) + \beta_k(x_n^{\mathrm{opt}}(k) - q_n(k))]^+$
  - where $q_n(k) = \mathrm{argmax}_{q \geq 0} b_n(k)q - f_n(q)$
- Send $b_n(k+1)$ to the ISO

Bid Adjustment Algorithm: convergence

Bids converge at a linear rate to neighborhood of the efficient Nash equilibrium

Theorem (Convergence)

Given $\alpha > 0$ *and* $0 < r < \|b(1) - b^*\|$ *such that*

$$\alpha \leq \beta_k \leq B(r),$$

there exists $\ell \in \mathbb{Z}_{\geq 1}$ *satisfying*

1. *for* $k < \ell$, $\|b(k) - b^*\| \geq r$ *with* $\|b(k+1) - b^*\| \leq \left(1 - \frac{\alpha}{2a_{\max}}\right)^{k/2} \|b(1) - b^*\|$
2. $\|b(\ell) - b^*\| < r$
3. *for* $k > \ell$, $\|b(k) - b^*\| \leq \left(1 + \frac{B(r)}{2a_{\max}}\right)^{1/2} r$

- trade-off between accuracy & rate

Stopping criteria

ISO's stopping criteria

for $\epsilon > 0$*, stop when* $\|b(k+1) - b(k)\| \leq \epsilon$

Guarantee

- either $\|b(k) - b^*\| > r$ and $k < \ell$ then

$$\|b(k) - b^*\| \leq \epsilon\Big(1 - \Big(1 - \frac{\alpha}{2a_{\max}}\Big)^{1/2}\Big)^{-1}$$

- or $\|b(k) - b^*\| \leq r$
- or $k > \ell$ in which case

$$\|b(k) - b^*\| \leq \Big(1 + \frac{B(r)}{2a_{\max}}\Big)^{1/2} r.$$

- ISO doesn't know r; its value depends on the stepsizes
- for small ϵ, the stopping criteria might never be met
- number of iterations unknown to generators - no long-term strategy

Recovering the DC-OPF solution

- need for a procedure: $b(k) \sim b^* \not\Longrightarrow x^{\text{opt}}(b(k)) \sim x^*$

Determine Generation

After Bid Adjustment Algorithm has stopped at $k \in \mathbb{Z}_{\geq 1}$.

- *[Step 1:]* ISO requests the production q_n^{offer} that the gen. is willing to produce at bid $b_n(k)$
- *[Step 2:]* ISO projects q^{offer} onto the feasibility set of the DC-OPF problem to get the dispatch q^{disp}

Proposition (Bound on dispatch from Determine Generation)

Let k be the iteration at which the Bid Adjustment Algorithm *stops with bids $b(k) \in \mathbb{R}^N_{\geq 0}$. The dispatch obtained from* Determine Generation *satisfies*

$$\|q^{\text{disp}} - x^*\| \leq \frac{\|b(k) - b^*\|}{2a_{\min}}$$

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Why would ISO-aggregators adopt this scheme?

1. robustness to perturbation
2. robustness to collusion among sets of generators

Robustness to perturbation

Algorithm subject to perturbations

$$
\begin{aligned}
b(k+1) &= [b(k) + \beta_k(x^{\mathrm{opt}}(k) - q(k)) + d(k)]^+ \\
x^{\mathrm{opt}}(k+1) &\in \mathrm{Sol}_{\mathrm{sopf}}(b(k+1)) \\
q(k+1) &= \mathrm{Sol}_{\mathrm{eff}}(b(k+1))
\end{aligned}
$$

Disturbances might arise e.g., from agents using different stepsizes

Proposition (Convergence retained under small perturbations)

Given $\|d(k)\| \leq \theta \|b(k) - b^*\|$ *for all* $k \in \mathbb{Z}_{\geq 1}$ *with* $0 \leq \theta < \frac{1}{6}\left(1 - \frac{\alpha}{2a_{\max}}\right)$*, there exists* $\ell \in \mathbb{Z}_{\geq 1}$ *satisfying*

1. *for* $k < \ell$, $\|b(k) - b^*\| \geq r$ *&*

   $$\|b(k+1) - b^*\| \leq \left(1 - \frac{\alpha}{2a_{\max}} + 2\theta + 4\theta^2\right)^{\frac{k}{2}} \|b(1) - b^*\|$$

2. $\|b(\ell) - b^*\| < r$

3. *for* $k > \ell$, $\|b(k) - b^*\| \leq \left(1 + \dfrac{B(r)}{2a_{\max}} + 2\theta + 4\theta^2\right)^{1/2} r$

Robustness to perturbation

Algorithm subject to perturbations

$$\begin{aligned} b(k+1) &= [b(k) + \beta_k(x^{\text{opt}}(k) - q(k)) + d(k)]^+ \\ x^{\text{opt}}(k+1) &\in \text{Sol}_{\text{sopf}}(b(k+1)) \\ q(k+1) &= \text{Sol}_{\text{eff}}(b(k+1)) \end{aligned}$$

Disturbances might arise e.g., from agents using different stepsizes

Proposition (Bounded disturbance implies bounded bids)

Let $\|d(k)\| \leq d_{\max}$ for all $k \in \mathbb{Z}_{\geq 1}$ and let $\theta \in \left(0, \frac{1}{6}\left(1 - \frac{\alpha}{2a_{\max}}\right)\right)$. Then, for all $k \in \mathbb{Z}_{\geq 1}$,

$$\|b(k) - b^*\| \leq \left(1 - \frac{\alpha}{2a_{\max}} + 2\theta + 4\theta^2\right)^{k/2} \|b(1) - b^*\| + G(r, \theta, d_{\max})$$

As a consequence, as $k \to \infty$, if $\|d(k)\| \to 0$, then

$$\max\left\{\|b(k) - b^*\|, \left(1 + \frac{B(r)}{2a_{\max}}\right)^{1/2} r\right\} \to \left(1 + \frac{B(r)}{2a_{\max}}\right)^{1/2} r$$

Robustness to collusion

Generators $\mathcal{J} \subset \{1, \ldots, N\}$ form a **collusion** if at each $k \in \mathbb{Z}_{\geq 1}$, each $n \in \mathcal{J}$,

1. **[share information]**: knows $\mathcal{I}_k := \{(b_r(t), x_r^{\mathrm{opt}}(t)) \mid r \in \mathcal{J}, t \in \{1, \ldots, k\}\}$
2. **[deviate]**: uses information $\mathcal{I}_k$ to decide next bid $b_n(k+1)$

Incentive to collude

$\mathcal{J}$ has incentive to collude if $\exists$ execution, generator $\tilde{n} \in \mathcal{J}$, and $\ell \in \mathbb{Z}_{\geq 1}$ s.t.

$$u_{\tilde{n}}(b_{\tilde{n}}(k), x_{\tilde{n}}^{\mathrm{opt}}(k)) > u_{\tilde{n}}^{\mathsf{max}}, \text{ for all } k \geq \ell$$

($u_{\tilde{n}}^{\mathsf{max}}$ is maximum ultimate utility under Bid Adjustment Algorithm*)*

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Proposition (Robustness to collusion)

Assume

1. *ISO always selects a vertex solution, and*
2. *at each generator bus there is at least one generator following* BID ADJUSTMENT ALGORITHM

Then, the rest of the generators do not have an incentive to collude.

Comments

- no individual generator has any incentive to deviate
- there could be other notions of incentive to collude

Another definition

Incentive to collude

$\mathcal{J}$ has incentive to collude if $\exists$ execution and $\ell \in \mathbb{Z}_{\geq 1}$ s.t.

$$\sum_{n \in \mathcal{J}} u_n(b_n(k), x_n^{\text{opt}}(k)) > u_{\mathcal{J}}^{\max}, \text{ for all } k \geq \ell$$

($u_{\mathcal{J}}^{\max}$ is maximum ultimate net utility of the group $\mathcal{J}$ under Bid Adjustment Algorithm*)*

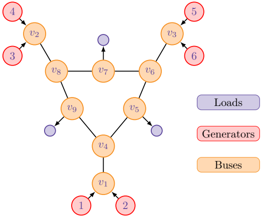
Proposition (Robustness to collusion)

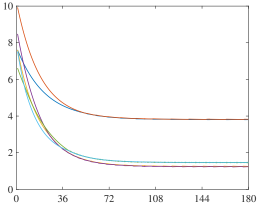
Assume that at each generator bus there is at least one generator that follows the update scheme of the Bid Adjustment Algorithm*, denote them $\mathcal{K} \subset \{1, \ldots, N\}$. Then, a group of generators $\mathcal{J} \subset \{1, \ldots, N\} \setminus \mathcal{K}$ have no incentive to collude if for all $\ell \in \mathbb{Z}_{\geq 1}$, there exists an integer $k_\ell \geq \ell$ for which*

$$\|b_{\{1,\ldots,N\}\setminus\mathcal{J}}(k_l) - b^*_{\{1,\ldots,N\}\setminus\mathcal{J}}\| \leq \frac{1}{\sqrt{1+|\mathcal{J}|}}\left(1 + \frac{B(r)}{2a_{\max}}\right)^{1/2} r$$

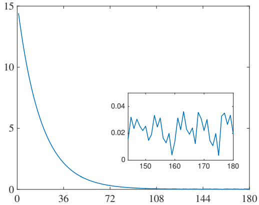
Simulation: IEEE 9-bus system

- cost function for each generator n is $f_n(x_n) = a_n x_n^2 + c_n x_n$
- all line capacities 2.5, except $\bar{z}_{56} = 1.5$, $\bar{z}_{36} = 3.0$, and $\bar{z}_{67} = 1.5$
- loads are $y_5 = 2$, $y_7 = 3$, and $y_9 = 1$





$b(k)$



$\|b(k) - b^*\|$

- constant stepsize, $\beta_k = 0.01$ for all k, and satisfy $\beta_k < 2a_n$
- linear convergence and ultimate bound

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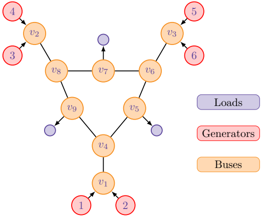

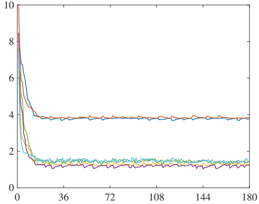

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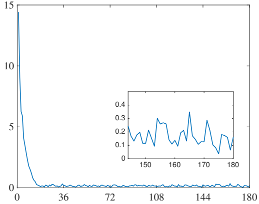

$\|b(k) - b^*\|$

- randomly selected stepsize from $[0.001, 0.1]$ (uniform distribution)
- bids still converge, but ultimate bound is larger

Simulation: IEEE 9-bus system

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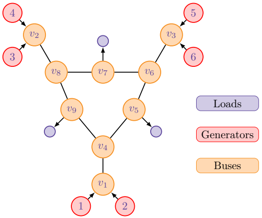

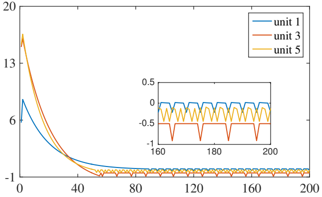

$u_n(b_n(k), x_n^{\mathrm{opt}}(k)) - u_n(b_n^*, x_n^*)$

- generators 1, 3, and 5 form a collusion
- stepsize is 0.01 for gens 2, 4, and 6

Summary

Conclusions

- electricity market game
- existence and uniqueness of the efficient NE
- dynamic analysis & viability of iterative bidding
- interconnection with frequency dynamics

A. Cherukuri & J. Cortés, To appear TNSE
T. Stegink, A. Cherukuri, J. Cortés, C. De Persis & A. van der Schaft, Submitted to TAC, 2018
T. Stegink, A. Cherukuri, J. Cortés, C. De Persis & A. van der Schaft, TCNS, 2019

Future work

- dynamics with supply function & capacity bidding
- other learning strategies – no regret learning