Stabilizing switching control of power converters: the lossy line and nonlinear case

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1. Port-Hamiltonian representation of power-converters
2. Controlled equilibria and Lyapunov functions
3. The transmission line as a port-Hamiltonian system
4. Power converter + transmission line + load

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Port-Hamiltonian representation of power converters

Example: *Boost converter*

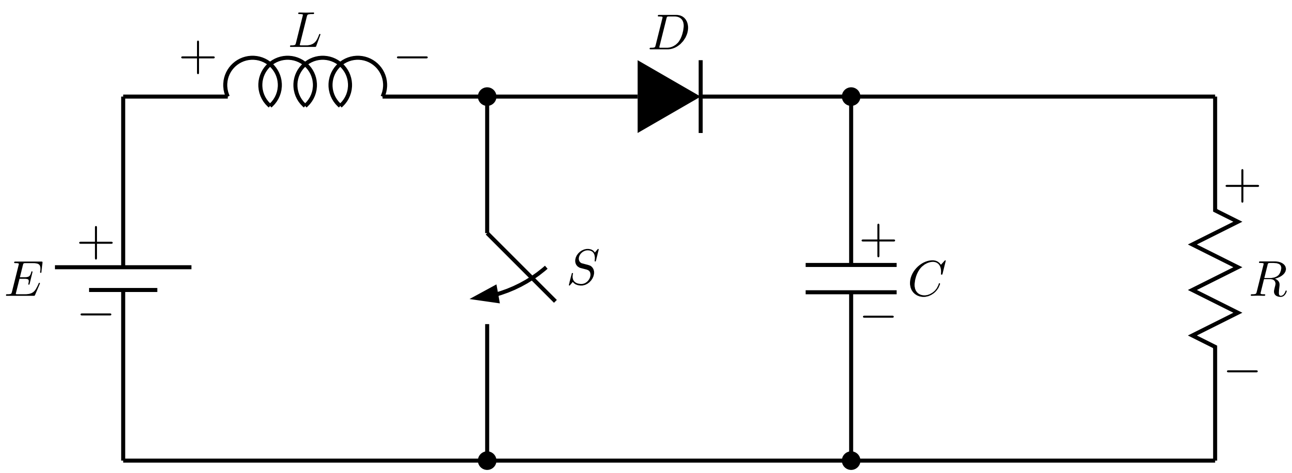

Figure 1: Boost circuit with clamping diode

The circuit consists of a capacitor C with electric charge q_C, an inductor L with magnetic flux linkage ϕ_L, and a resistive load R, together with an ideal diode and an ideal switch S, with switch positions $s = 1$ (switch closed) and $s = 0$ (switch open).

The voltage-current characteristics of the ideal diode and switch are depicted in Figure 2.

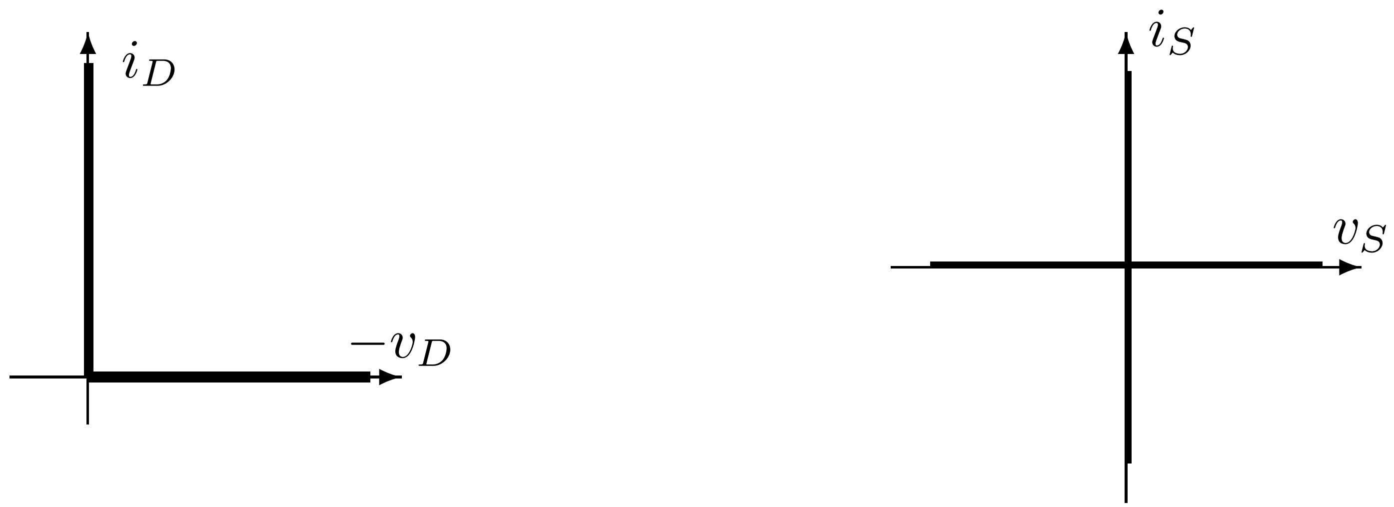

Figure 2: Voltage-current characteristic of an ideal diode and ideal switch

The ideal diode thus satisfies the complementarity conditions:

$$v_D i_D = 0, \quad v_D \leq 0, \quad i_D \geq 0.$$

This yields the port-Hamiltonian model (with $H(q_C, \phi_L) = \frac{1}{2C} q_C^2 + \frac{1}{2L}\phi_L^2$):

$$\begin{bmatrix} \dot{q}_C \\ \dot{\phi}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{R} & 1-s \\ s-1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q_C} = \frac{q_C}{C} \\ \frac{\partial H}{\partial \phi_L} = \frac{\phi_L}{L} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} E + \begin{bmatrix} s i_D \\ (s-1) v_D \end{bmatrix}$$

$$I = \frac{\phi_L}{L}$$

By disregarding the 'discontinuous modes' (*normal operation*), that is, assuming that when the switch is closed ($s = 1$) the diode is open ($i_D = 0$), while if the switch is open ($s = 0$) the diode is closed ($v_D = 0$), we obtain the *switching port-Hamiltonian system*

$$\begin{bmatrix} \dot{q}_C \\ \dot{\phi}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{R} & 1-s \\ s-1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q_C} = \frac{q_C}{C} \\ \frac{\partial H}{\partial \phi_L} = \frac{\phi_L}{L} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} E$$

$$I = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q_C} = \frac{q_C}{C} \\ \frac{\partial H}{\partial \phi_L} = \frac{\phi_L}{L} \end{bmatrix} = \frac{\phi_L}{L}$$

In general, a switching port-Hamiltonian system without algebraic equality and inequality constraints is defined as

$$\begin{aligned} \dot{x} &= F(\rho)z + g(\rho)u, \qquad z = \tfrac{\partial H}{\partial x}(x) \\ y &= g^T(\rho)z \end{aligned}$$

with

$$F(\rho) = J(\rho) - R(\rho), \qquad J(\rho) = -J^T(\rho), \qquad R(\rho) = R^T(\rho) \geq 0$$

Note that the system is passive for every switching sequence:

$$\frac{d}{dt}H = -\frac{\partial^T H}{\partial x}(x)R(\rho)\frac{\partial H}{\partial x}(x) + u^T y \leq u^T y$$

Basic property of power converters in normal operation

Consider the port-Hamiltonian representation

$$\begin{aligned} \dot{x} &= F(\rho)z + g(\rho)E + g_l u, \qquad z = \frac{\partial H_p}{\partial x}(x) \\ y &= g_l^T z \end{aligned}$$

with vector of Boolean variables $\rho \in \{0,1\}^p$, $H_p(x)$ the total stored electromagnetic energy, and (u, y) the current-voltage pair over the resistive load.

Let x_0 be an equilibrium of the *averaged* model, that is

$$F(\rho_0)z_0 + g(\rho_0)E + g_l u_0 = 0, \qquad z_0 = \frac{\partial H_p}{\partial x}(x_0)$$

for some $\rho_0 \in [0,1]^p$ and u_0. Then

$$\begin{aligned}
\dot{x} &= F(\rho)(z - z_0) + F(\rho)z_0 + g(\rho)E + g_l u \\
&= F(\rho)(z - z_0) + [F(\rho) - F(\rho_0)]z_0 + [g(\rho) - g(\rho_0)]E + g_l(u - u_0) \\
&\quad + F(\rho_0)z_0 + g(\rho_0)E + g_l u_0 \\
&= F(\rho)(z - z_0) + [F(\rho) - F(\rho_0)]z_0 + [g(\rho) - g(\rho_0)]E + g_l(u - u_0)
\end{aligned}$$

For many power converters in normal operation we know that

$$\begin{aligned} F(\rho) - F(\rho_0) &= \textstyle\sum_{i=1}^{p} F_i(\rho_i - \rho_{0i}) \\ g(\rho) - g(\rho_0) &= \textstyle\sum_{i=1}^{p} g_i(\rho_i - \rho_{0i}) \end{aligned}$$

and thus

$$\dot{x} = F(\rho)(z - z_0) + \sum_{i=1}^{p} [F_i z_0 + g_i E](\rho_i - \rho_{0i}) + g_l(u - u_0)$$

Take as Lyapunov/storage function

$$V(x) := H_p(x) - (x - x_0)^T \frac{\partial H_p}{\partial x}(x_0) - H_p(x_0)$$

Then

$$\begin{aligned} \tfrac{d}{dt}V(x) &= [\tfrac{\partial H_p}{\partial x}(x) - \tfrac{\partial H_p}{\partial x}(x_0)]^T \dot{x} = (z - z_0)^T \dot{x} = (z - z_0)^T F(\rho)(z - z_0) \\ &\quad + \textstyle\sum_{i=1}^{p} (z - z_0)^T [F_i z_0 + g_i E](\rho_i - \rho_{0i}) + (z - z_0)^T g_l(u - u_0) \end{aligned}$$

with $(z - z_0)^T F(\rho)(z - z_0) = -(z - z_0)^T R(\rho)(z - z_0) \leq 0.$

Thus at any time we can choose the Boolean variables $\rho_i \in \{0, 1\}$ in such a manner that

$$\frac{d}{dt}V(x) \leq (z - z_0)^T g_l(u - u_0)$$

implying passivity of the switched system with respect to the input vector $u - u_0$ and output vector $y - y_0 = g_l^T(z - z_0)$.

As a consequence, if the converter is terminated on a resistive load R (and hence the equilibrium x_0 should be such that $y_0 = -Ru_0$) then the switched converter is generally asymptotically stable around x_0. Thus the voltage over the resistive load can be stabilized around any set-point. (For the linear case, cf. Buisson, Cormerais, Richard.)

Remark 1 *Note that for linear capacitors and inductors we have*

$$H_p(x) = \frac{1}{2}x^T Q x, \quad V(x) = \frac{1}{2}(x - x_0)^T Q(x - x_0)$$

Power converter connected to the load via transmission line

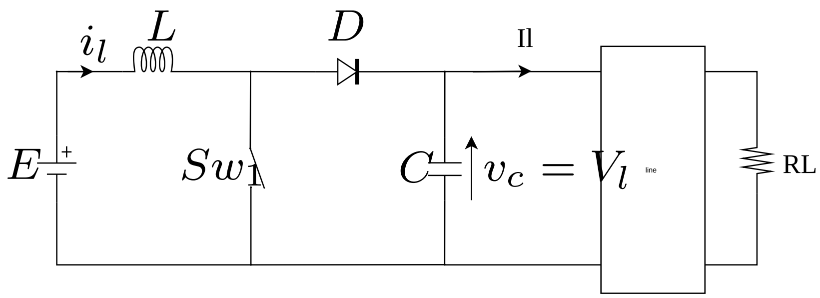

Figure 3: The Boost converter with a transmission line

Port-Hamiltonian representation of the transmission line

Let $s \in [0,1]$ be the spatial variable. The energy variables are the charge density $Q = Q\,(t,s)\,ds$, and the flux density $\varphi = \varphi\,(t,s)\,ds$. The total energy stored in the transmission line is

$$H_l\,(Q,\varphi) = \int_0^1 \frac{1}{2}\left(\frac{Q^2\,(t,s)}{C_l} + \frac{\varphi^2\,(t,s)}{L_l}\right) ds$$

where C_l and L_l are the distributed capacitance and inductance of the line. The voltage and current at any position $s \in [0,1]$ are

$$V\,(t,s) = \frac{Q\,(t,s)}{C_l}$$

$$I\,(t,s) = \frac{\varphi\,(t,s)}{L_l}$$

The lossy transmission line is given by the telegrapher's equations

$$\frac{\partial Q}{\partial t} = -\frac{\partial I}{\partial s} - G_l V\left(s,t\right)$$
$$\frac{\partial \varphi}{\partial t} = -\frac{\partial V}{\partial s} - R_l I\left(s,t\right),$$

where G_l and R_l are the distributed conductance and the distributed resistance of the line. Since the transmission line is terminated on a resistive load R_L

$$V\left(t,1\right) = R_L I\left(t,1\right)$$

while $V\left(t,0\right)$ and $I\left(t,0\right)$ is the voltage, respectively the current, at the beginning of the line, as to be connected to the voltage=current pair (y,u) of the power converter.

This is a port-Hamiltonian system satisfying

$$\frac{d}{dt}H_l \leq -R_L V^2(t,1) + uy$$

Power converter + transmission line + load

The total system is a port-Hamiltonian system with Hamiltonian $H_p + H_l$.

An equilibrium $(V_0(s), I_0(s))$ for the line satisfies

$$\frac{\partial Q}{\partial t} = \frac{\partial \varphi}{\partial t} = 0,$$

and thus is a solution of

$$\begin{pmatrix} \frac{\partial V_0}{\partial s}(s) \\ \frac{\partial I_0}{\partial s}(s) \end{pmatrix} = \begin{pmatrix} 0 & -R_l \\ -G_l & 0 \end{pmatrix} \begin{pmatrix} V_0(s) \\ I_0(s) \end{pmatrix},$$

satisfying additionally the boundary condition

$$\begin{aligned} V_0(0) &= y_0 \\ V_0(0) &= u_0 \\ V_0(1) &= R_L I_0(1) \end{aligned}$$

An equilibrium of the system *power converter + transmission line + load* is thus given by (z_0, V_0, I_0) linked by certain u_0, y_0 (the equilibrium current and voltage at the left-end of the line).

Take as Lyapunov function of the total system

$$\begin{aligned} V(x,Q,\phi) &:= H(x) - (x - x_0)^T \tfrac{\partial H}{\partial x}(x_0) - H(x_0) \\ &+ H_l(Q - Q_0, \phi - \phi_0) \end{aligned}$$

It follows that

$$\begin{aligned} \tfrac{d}{dt} V &= (z - z_0)^T F(\rho)(z - z_0) + \textstyle\sum_{i=1}^{p} (z - z_0)^T [F_i z_0 + g_i E](\rho_i - \rho_{0i}) \\ & -(V(t,1) - V_0)^2 \Big/ R_L - \int_0^1 \Big[(V - V_0)^2 G_l + (I - I_0)^2 R_l \Big] ds \end{aligned}$$

and, as before, we can make the right-hand side negative by choice of the Boolean variables $\rho_i \in \{0, 1\}$ (depending only on the values of the state variables of the power converter).

Conclusions

1. Stabilizing switching control strategy for linear power converters has been extended to
   - Lossy transmission line between power converter and resistive load.
   - Nonlinear capacitors and inductors.
2. How to extend to nonlinear resistive elements ?
3. How to deal with discontinuous modes ?