

Reciprocals of well-posed linear systems

During the past decade great progress has been made in studying the class of well-posed linear systems. Roughly speaking, these are generalizations of the finite-dimensional systems specified by 4 matrices A, B, C, D to systems with Hilbert spaces as the input, output and state spaces and unbounded generating operators A, B, C . The interest in this generalization is motivated by control and system theoretic problems for systems described by delay equations and partial differential equations. Although the unbounded nature of the generating operators introduces many technical complications, a mature and relatively complete theory has been developed (see Staffans [5]). As would be expected, not all the results are as nice as the ones for finite-dimensional systems. In particular, the Riccati equation theory involves awkward assumptions that cannot be easily verified (see Mikkola [4]).

This project proposes a novel approach to studying a well-posed linear system with generating operators A, B, C , and transfer function \mathbf{G} and under the generic assumption that $0 \in \rho(A)$. We introduce the associated reciprocal system $A^{-1}, A^{-1}B, -A^{-1}C, \mathbf{G}(0)$ which has four BOUNDED generating operators. There are nice relationships between the system theoretic properties of a well-posed linear system and its reciprocal system. For example, they have the same input, output and input-output stability properties. Moreover, many control and system theoretic problems for well-posed linear systems can be translated into equivalent problems for their reciprocal systems. Due to the bounded nature of the generators, the problems for the reciprocal system are easier to solve and these solutions can be translated back to solutions for the original well-posed linear system.

This approach has proved successful in solving spectral factorization problems and optimal control problems that have solutions in terms of a Riccati equation. Current research is being carried out on the existence of (pseudo-) coprime factorizations, numerical solutions of Riccati equations and control problems for unstable systems.

Historical Remarks I have had some enlightening correspondence with Olof Staffans, Piotr Grabowski, Luciano Pandolfi and Rien Kaashoek about my concept of reciprocal systems. Like many ideas, it does not appear to be completely new. According to Olof it can be traced back to Livsic in his book [3], and indeed, although the treatment of systems there is rather different, the reciprocal idea is present. According to Piotr and Luciano, the relationship A, C with $A^{-1}, A^{-1}C$ has been used by Fattorini and Triggiani in the study of controllability for boundary control systems. A similar idea can be found in Curtain and Pritchard [2, Theorem 3.15]. Rien has told me of an even the closer connection in the study of certain Wiener-Hopf problems, where the reciprocal of a transfer function is used to make the analysis simpler. I first became interested in these ideas during a talk by Piotr on Lyapunov stability in 2001. At about the same time Zwart and Guo were investigating the relationship between the stability properties of A (see their paper in the Proceedings of the MTNS 2002). Apparently Piotr Grabowski and Frank Callier are using a reciprocal system concept in their work on the circle criterion for boundary control via Lyapunov stability and Lur'e equations

(see [1]). I am interested in any other information on other applications of similar ideas.

References.

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4. K. Mikkola. *Infinite-Dimensional Linear Systems, Optimal Control and Riccati Equations*. PhD thesis, Helsinki University of Technology, October 2002.
5. O.J. Staffans. Well-Posed Linear Systems I: General Theory. Manuscript. Draft available at <http://www.abo.fi/~staffans/publ.htm>