strips and strings: Möbius' models unveiled

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&

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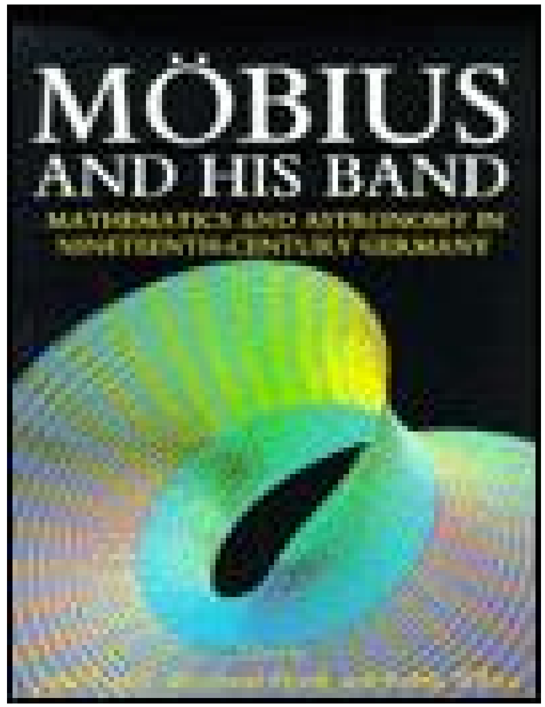
Möbius

August Ferdinand Möbius (1790–1868)



John Fauvel, Robin Wilson, Raymond Flood (eds.), 1993.

comparison.

- Leipzig: tenure in 1816; full professor in 1844 (age: > 50)
- **A**ugust Ferdinand **M**öbius $\leftrightarrow$ Johannes **A**rnoldus van **M**aanen
- didactically well-written papers

-  $\leftrightarrow$ 

- sharp contrast(!!): many/no female students möbius strip/band/ring

möbius transformation

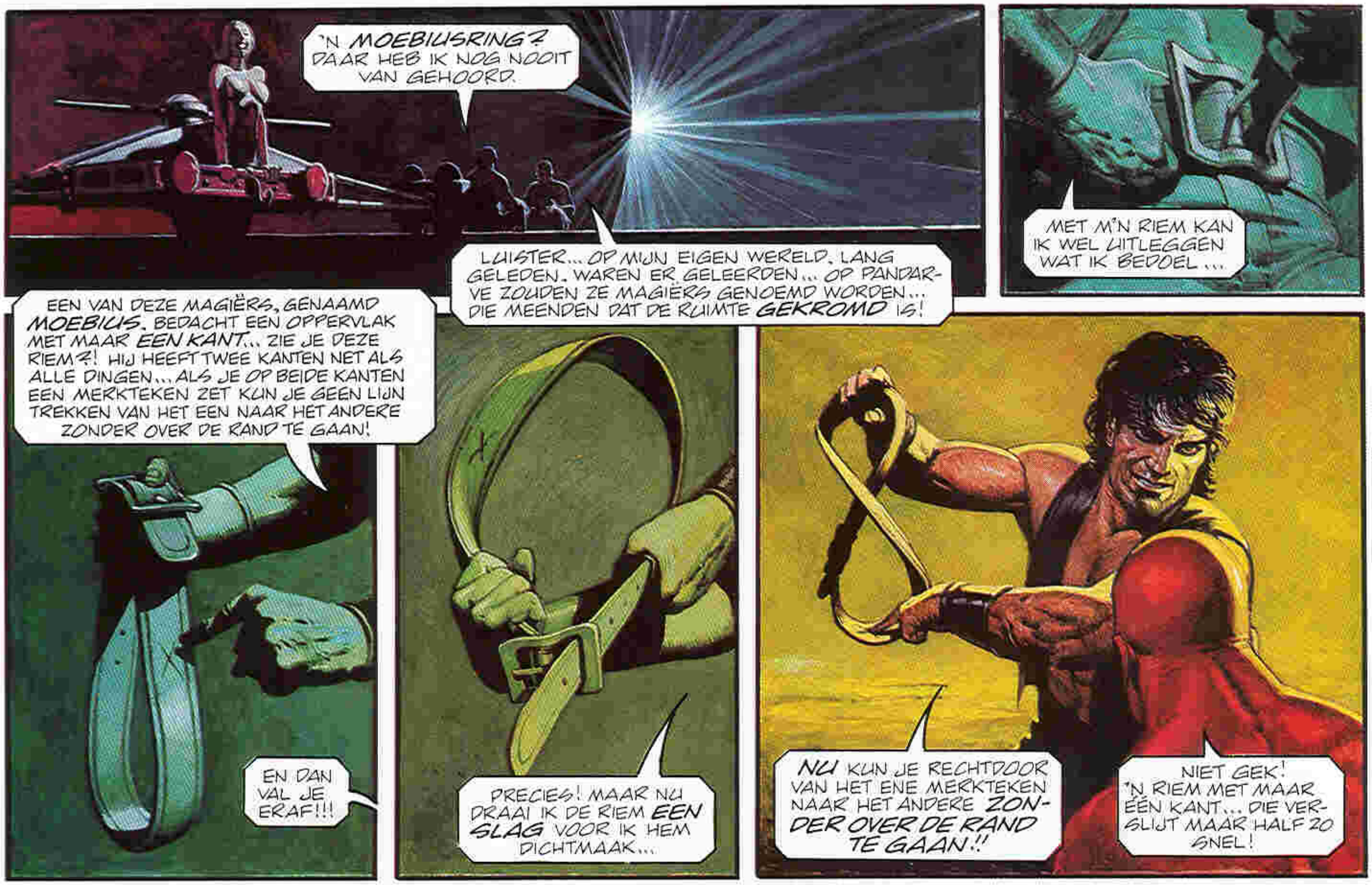
möbius function

möbius inversion

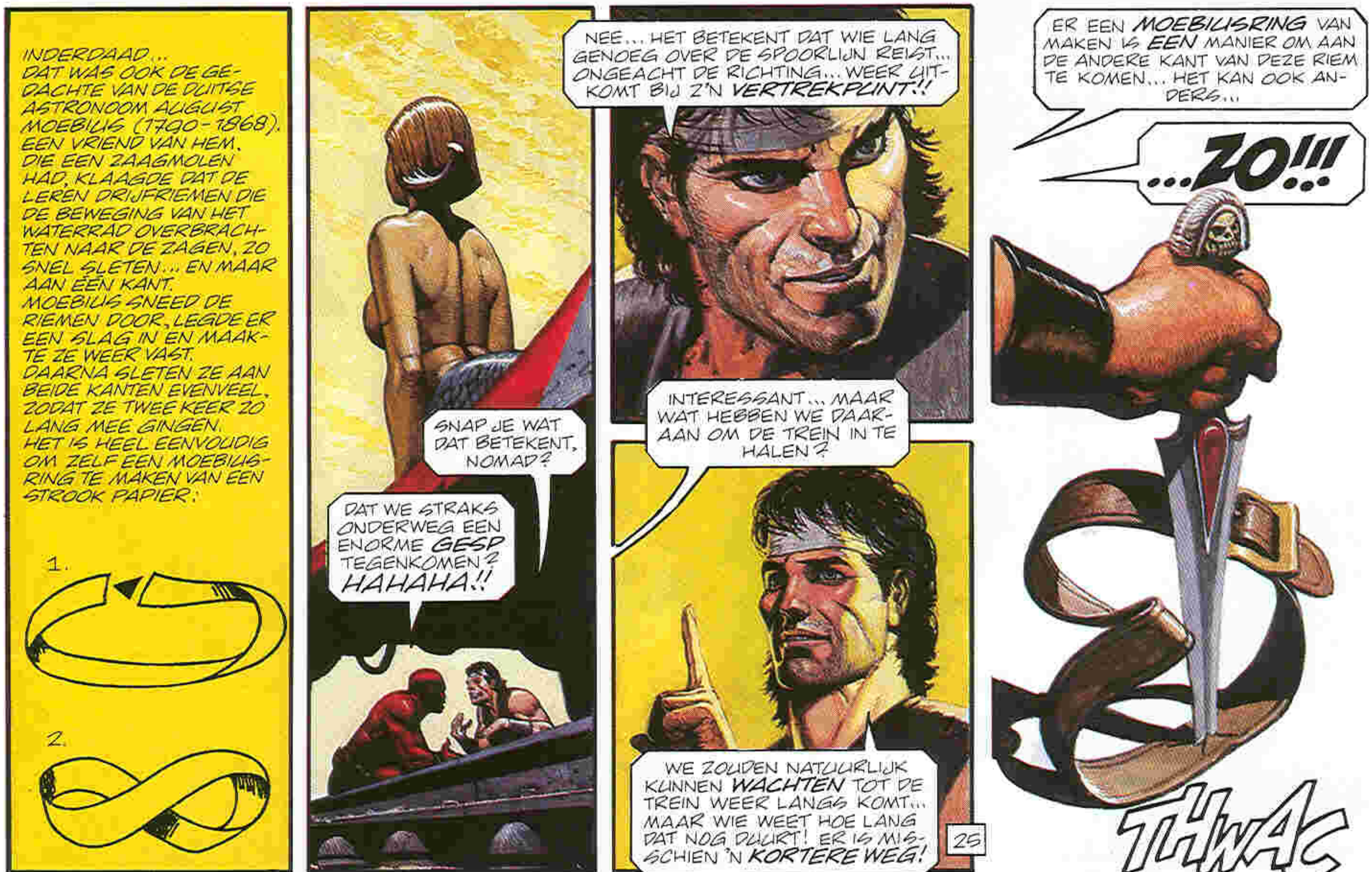
new: möbius string

strip

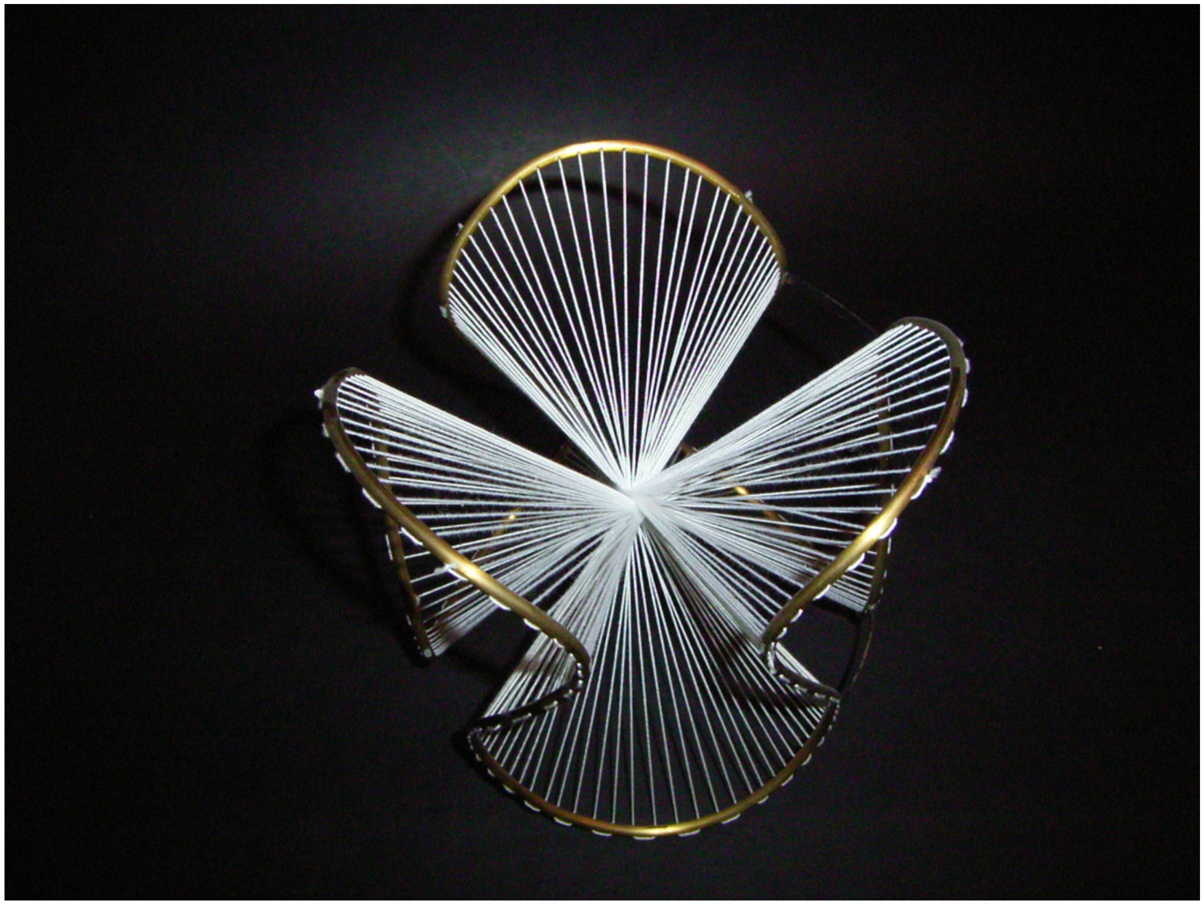
to strip $\leftrightarrow$ strip/band $\leftrightarrow$ strip/cartoon



Don Lawrence & Martin Lodewijk: De Wentelwereld


INDERDAAD... DAT WAS OOK DE GEDACHTE VAN DE DUITSE ASTRONOOM AUGUST MOEBIUS (1790-1868). EEN VRIEND VAN HEM, DIE EEN ZAAGMOLEN HAD, KLAAGDE DAT DE LEREN DRIJFRIEMEN DIE DE BEWEGING VAN HET WATERRAD OVERBRACHTEN NAAR DE ZAGEN, ZO SNEL SLETEN... EN MAAR AAN EEN KANT. MOEBIUS SNEED DE RIEMEN DOOR, LEGDE ER EEN SLAG IN EN MAAKTE ZE WEER VAST. DAARNA SLETEN ZE AAN BEIDE KANTEN EVENVEEL, ZODAT ZE TWEE KEER ZO LANG MEE GINGEN. HET IS HEEL EENVOUDIG OM ZELF EEN MOEBIUSRING TE MAKEN VAN EEN STROOK PAPIER:
1.
2.
SNAP JE WAT DAT BETEKENT, NOMAD?
DAT WE STRAKS ONDERWEG EEN ENORME **GESP** TEGENKOMEN? **HAHAHA!!**
NEE... HET BETEKENT DAT WIE LANG GENOEG OVER DE SPOORLIJN REIST... ONGEACHT DE RICHTING... WEER UITKOMT BIJ Z'N **VERTREKPUNT!!**
INTERESSANT... MAAR WAT HEBBEN WE DAARAAN OM DE TREIN IN TE HALEN?
WE ZOUDEN NATUURLIJK KUNNEN **WACHTEN** TOT DE TREIN WEER LANGS KOMT... MAAR WIE WEET HOE LANG DAT NOG DUURT! ER IS MISSCHIEN 'N **KORTERE WEG!**
25
ER EEN **MOEBIUSRING** VAN MAKEN IS **EEN** MANIER OM AAN DE ANDERE KANT VAN DEZE RIEM TE KOMEN... HET KAN OOK ANDERS...
...ZO!!!
THWAC

string



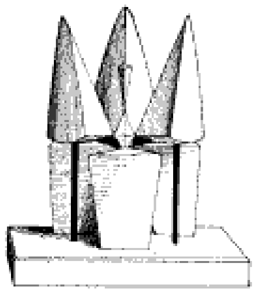
7 string models,

produced in 1899 by the company Martin Schilling (Leipzig),

extending earlier collections of models of company Ludwig Brill (Darmstadt).

Models for the Higher Mathematical Instruction

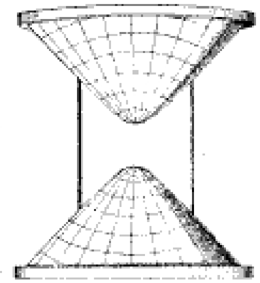
PUBLISHED BY L. BRILL IN DARMSTADT (GERMANY).



MODELS

Of Plaster, constructed of Silk Threads in Brass Frames, of Wire, Sheet-Brass, etc.

16 SERIES.



The models of seven of those series are constructed after the originals in the Mathematical Institute of the Royal Polytechnicum in Munich, under the direction of Prof. Dr. Brill, Prof. Dr. Klein and Prof. Dr. Dyck. Other series of Prof. Dr. Kummer in Berlin, Prof. Dr. Neovius in Helsingfors, Prof. Dr. Rodenberg in Hannover, Prof. Dr. Rohn in Dresden, Dr. Schlegel in Hagen, Prof. Dr. Wiener in Karlsruhe, Privat-Docent Dr. Wiener in Halle, etc.

Excepting two series, all the models can be obtained separately. An explanatory text accompanies most of them. The prices are exclusive of packing and transportation.

Prospectus furnished, if desired, gratis and postpaid. Of the whole 217 numbers of the collection, 158 are of plaster, 19 are constructed of silk threads, 40 of wire, etc. They refer to almost all the departments of mathematical knowledge: synthetical and analytical geometry, theory of curvature, mathematical physics, theory of functions, etc.

juli 1890

designer of the Möbius strings:

Hermann Wiener (Darmstadt, 1857–1939)

earlier design, on two plaster balls:

Alexander Wilhelm von Brill (1842–1935)

(brother of Ludwig)

Brill models, Serie XVII 2a & 2b (1886)

Problem:

real constants a, b, c

function $x \mapsto y = \sqrt{x^3 + ax^2 + bx + c}$

what possible graphs?

ways to visualize these graphs:

include graph of $x \mapsto y = -\sqrt{x^3 + ax^2 + bx + c}$ as well,

so consider points (x, y) satisfying

$$y^2 = x^3 + ax^2 + bx + c.$$

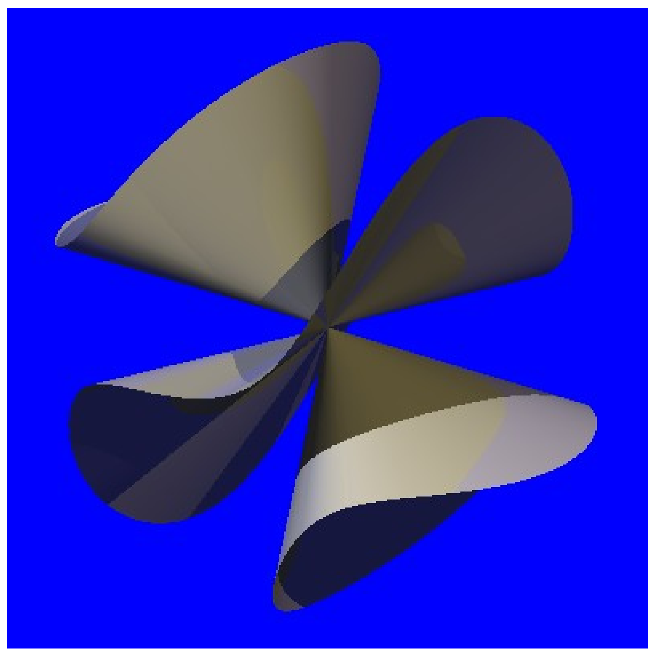
Any such point (x, y) determines a line ℓ in $\mathbb{R}^3$,

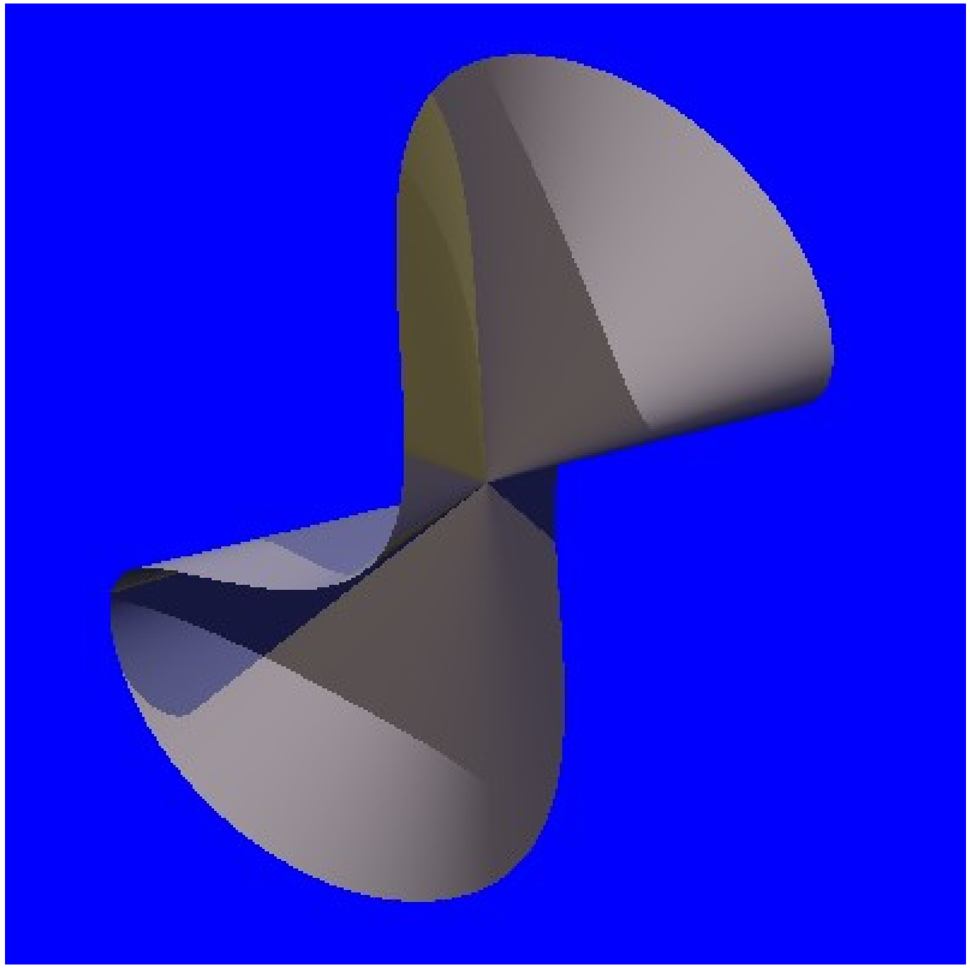
namely the line through $(0, 0, 0)$ and $(x, y, 1)$.

union of these lines ℓ:

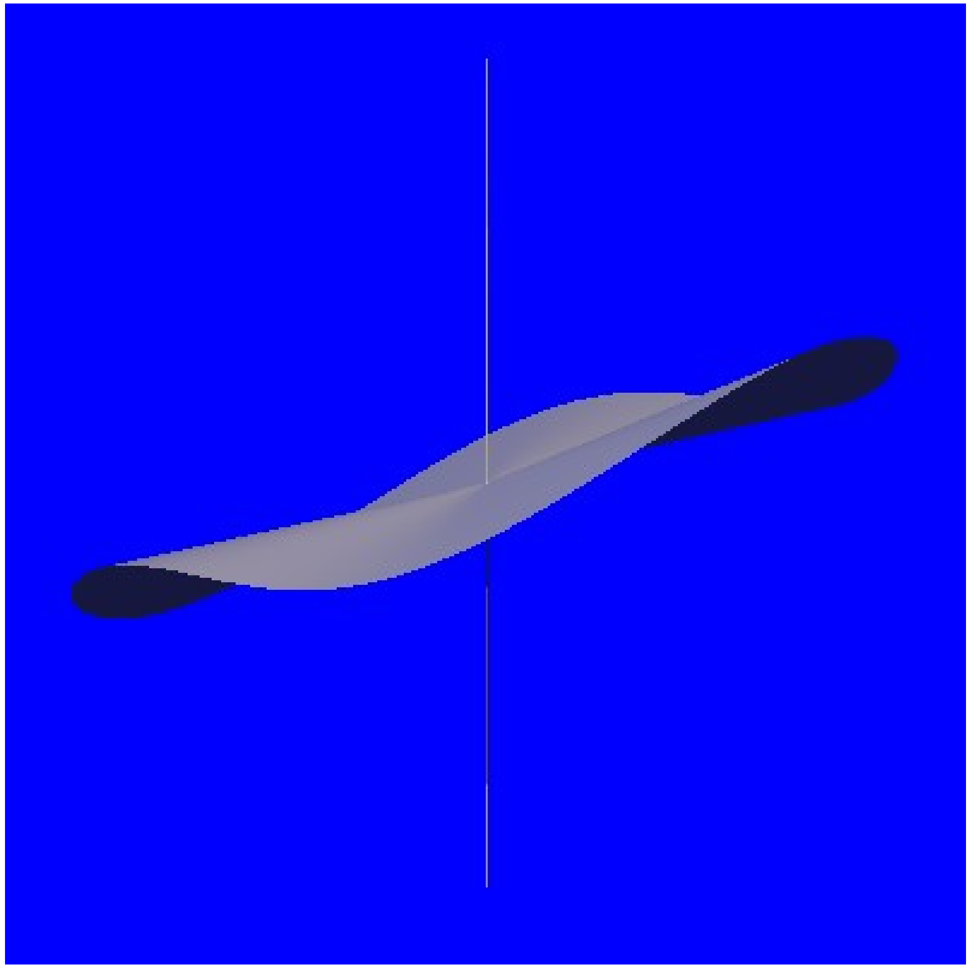
a cone through $(0,0,0)$, with equation $y^2 z = x^3 + ax^2 z + bxz^2 + cz^3$.

Example: $\sqrt{x^3 + 2x^2 - 2x}$ yields the cone

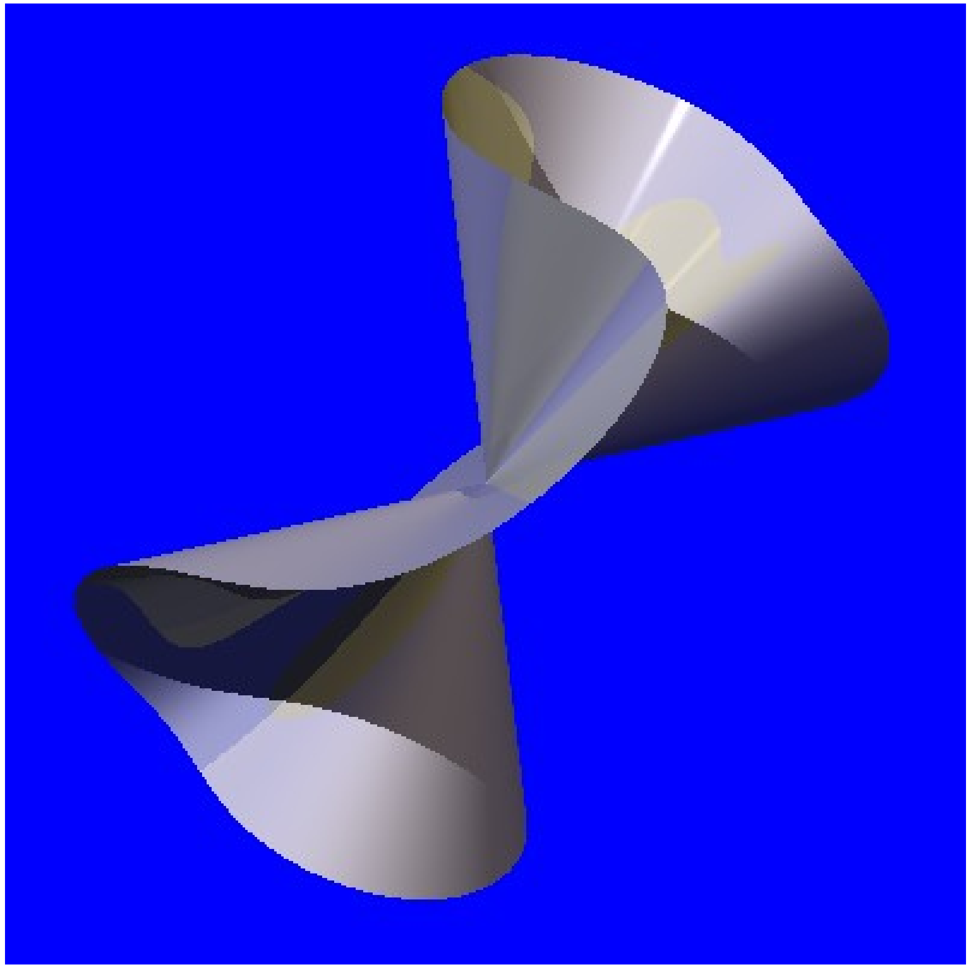




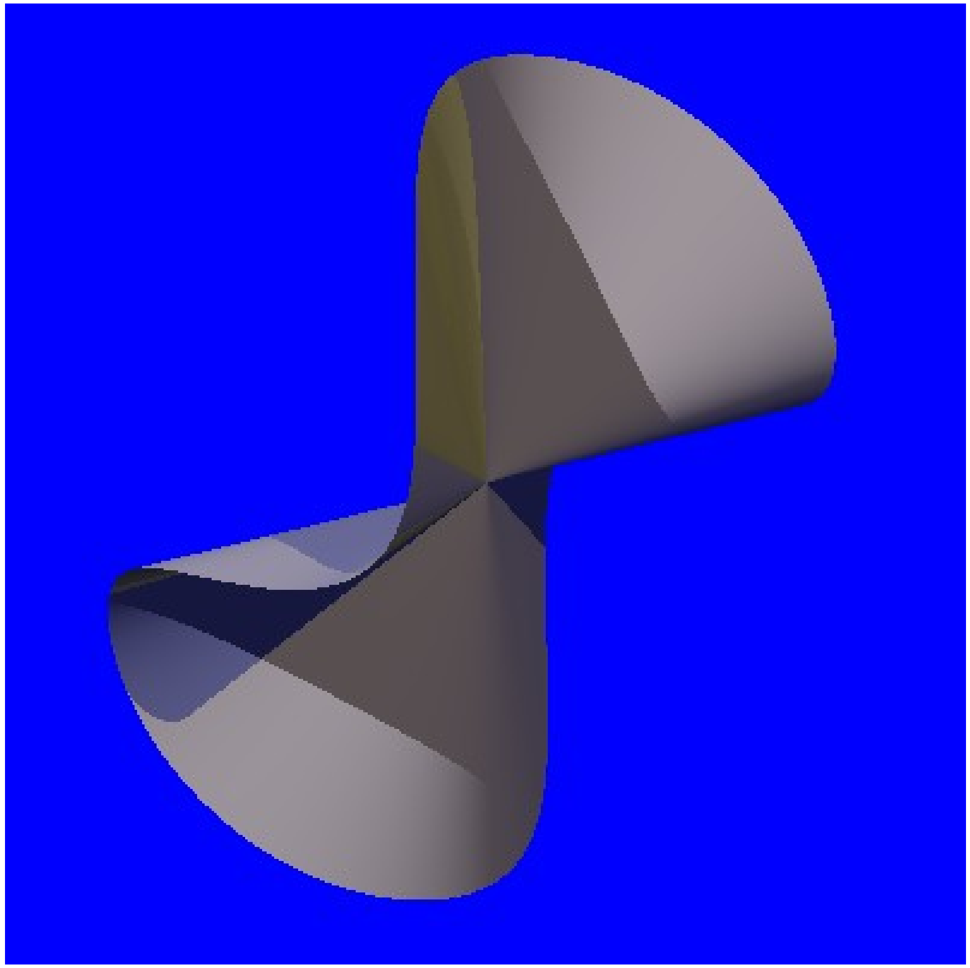
$$\sqrt{x^3 + 2x^2 + 2x}$$



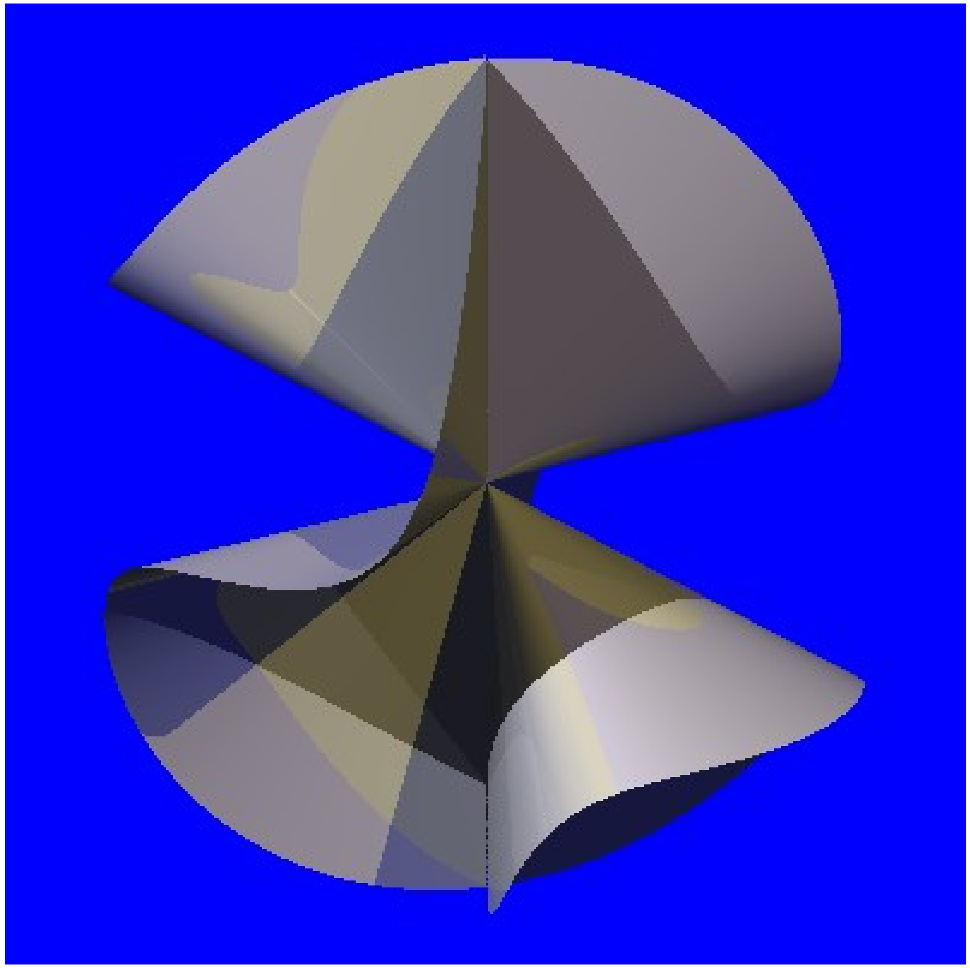
$$\sqrt{x^3 - 2x^2}$$



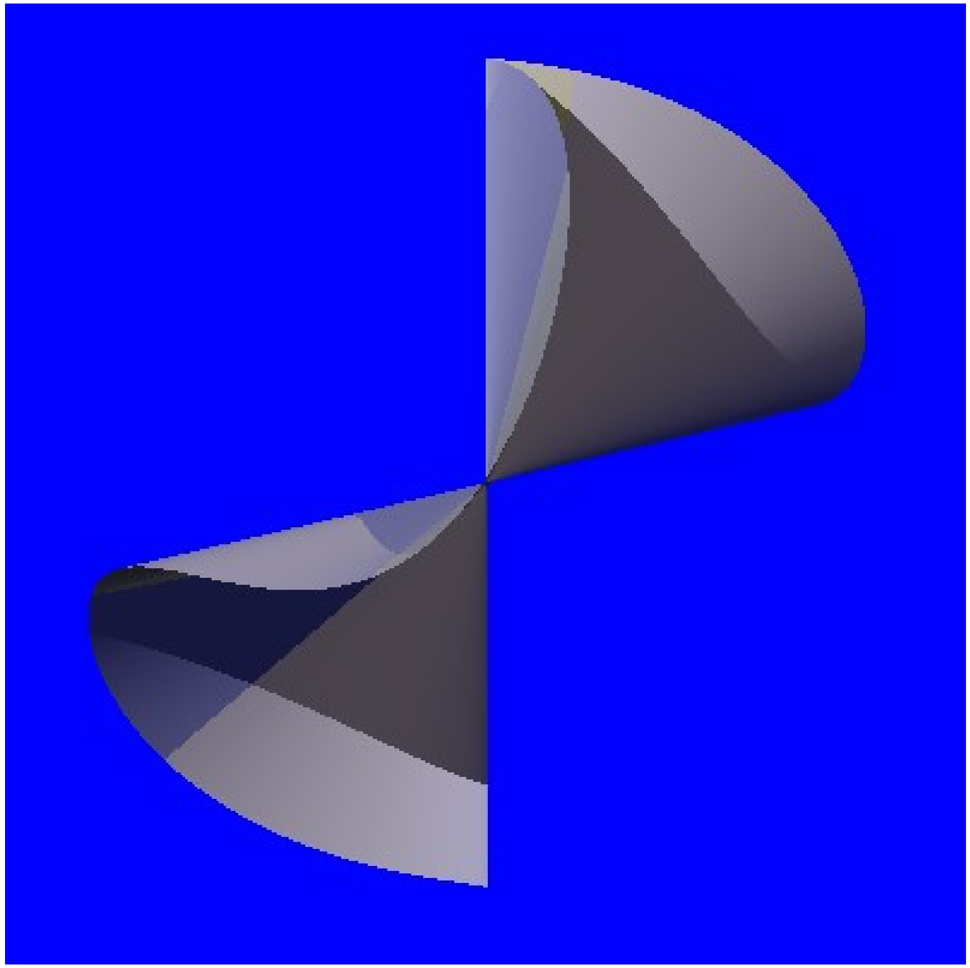
$$\sqrt{x^3 - 2x^2 + \frac{5}{4}x}$$



$$\sqrt{x^3 + 2x^2 + \frac{4}{3}x}$$



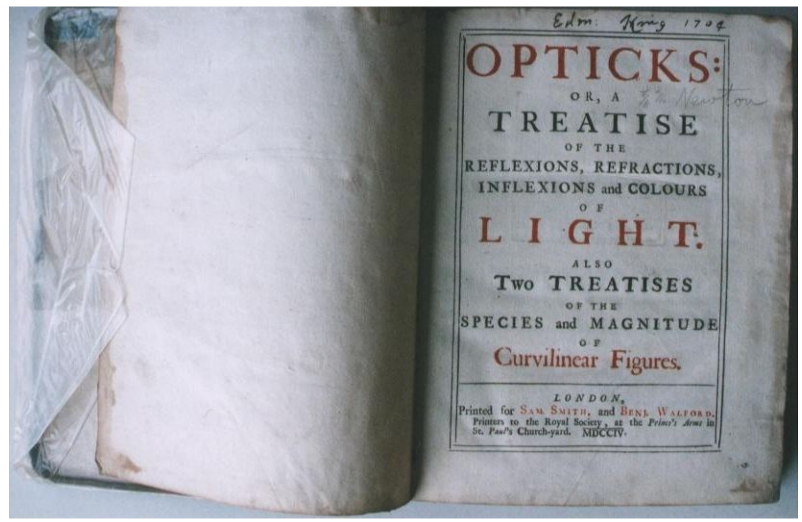
$$\sqrt{x^3 + 2x^2}$$



$\sqrt{x^3}$

Theory starts with Sir Isaac Newton (1643–1727)

Appendix *Enumeratio Linearum Tertii Ordinis* to book *Opticks* (1704)



Newton: 5 types according to roots of

$$p(x) := x^3 + ax^2 + bx + c = 0 :$$

- one triple root. Graphs of $\pm\sqrt{p(x)}$:

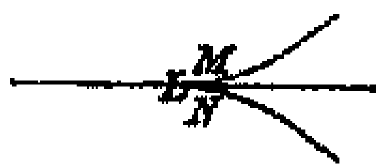


Fig. 11.

parabola cuspidata

• one double root and one simple, larger root. Graphs:

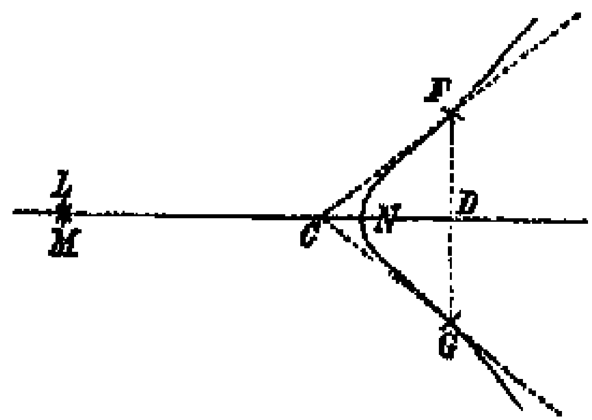


Fig. 8.

parabola punctata

• one double root and one simple, smaller root. Graphs:

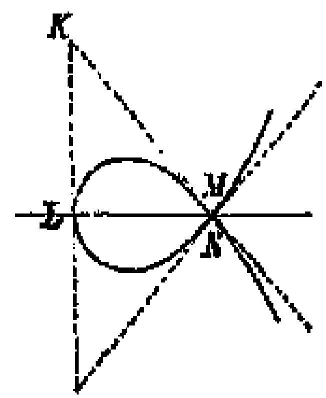


Fig. 10.

parabola nodata

• only one real root, which is simple. Graphs:

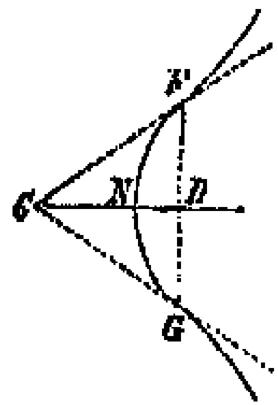


Fig. 9a.

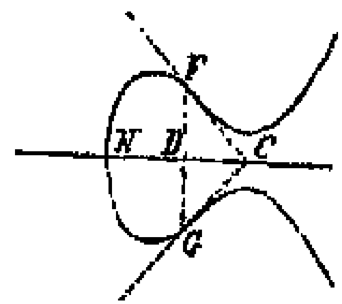


Fig. 9b.

parabola pura

• three distinct roots. Graphs:

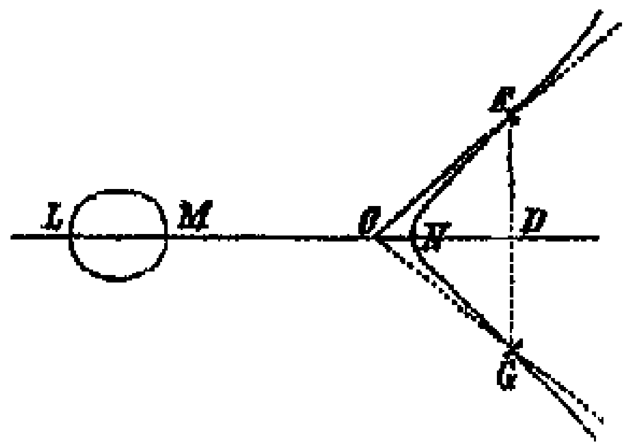


Fig. 7.

parabola campaniformis cum ovali

Möbius: *Ueber die Grundformen der Linien der dritten Ordnung*

(82 pages, published in 1852).

Def. A flex point of $\sqrt{p(x)}$ is a point of the graph where the tangent line meets with multiplicity ≥ 3.

Thm. $\sqrt{x^3 + ax^2 + bx + c}$ has:

- no flex point for the parabola cuspidata and nodata;

- precisely one flex point for the three other cases.

.

Idea for a modern proof: the third derivative of the function is positive on the domain.

Thm.

- for the parabola punctata and campaniformis cum ovali, the slope of the tangent line at the flex point is positive.

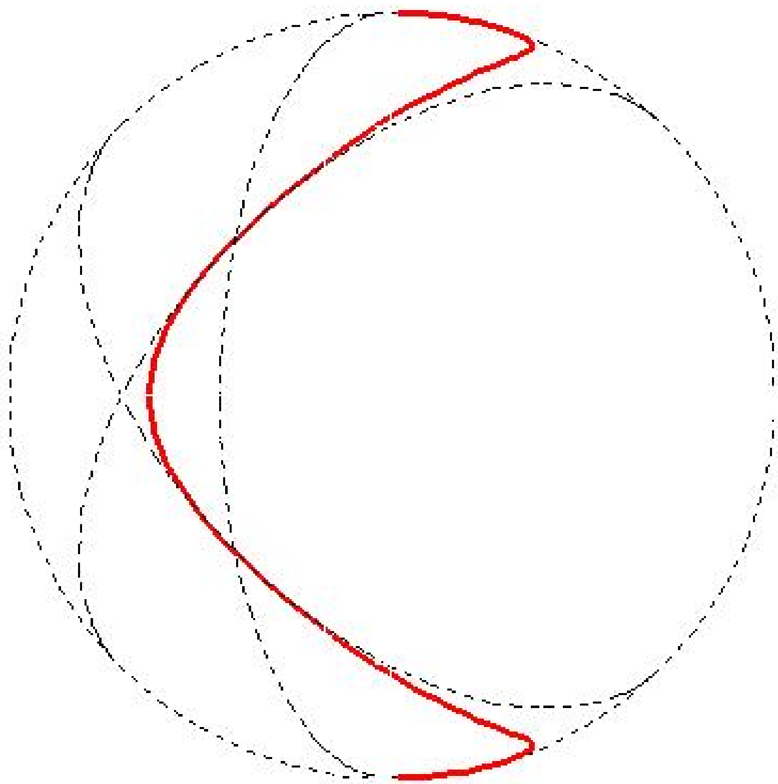
- There exist three different types of the parabola pura, depending on the slope of the tangent line at the flex being negative, zero, or positive.

Hence [Möbius]: there are in total 7 different types of graphs!

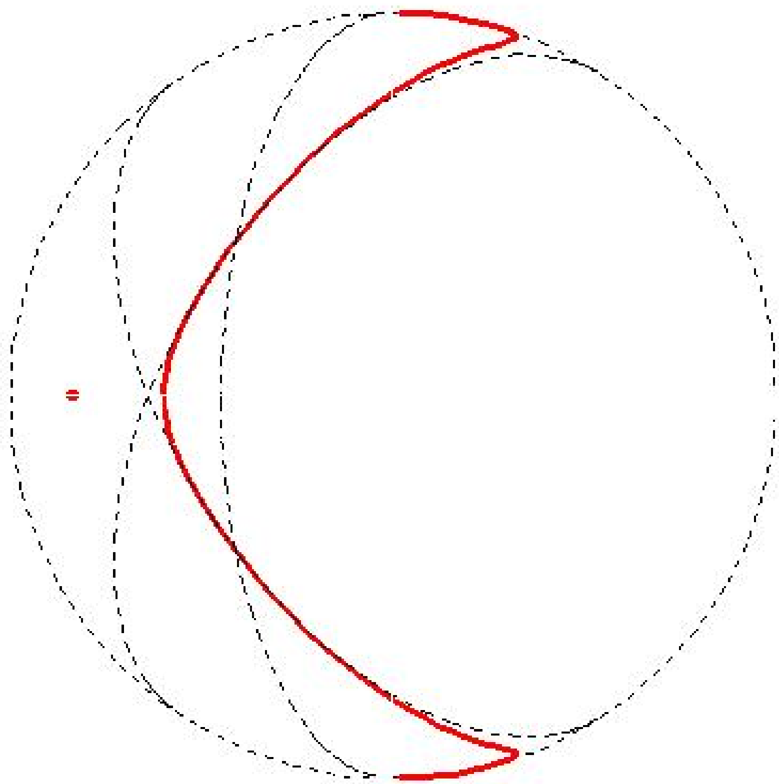
Intersecting the cone given by

$$y^2 z = x^3 + ax^2 z + bxz^2 + cz^3$$

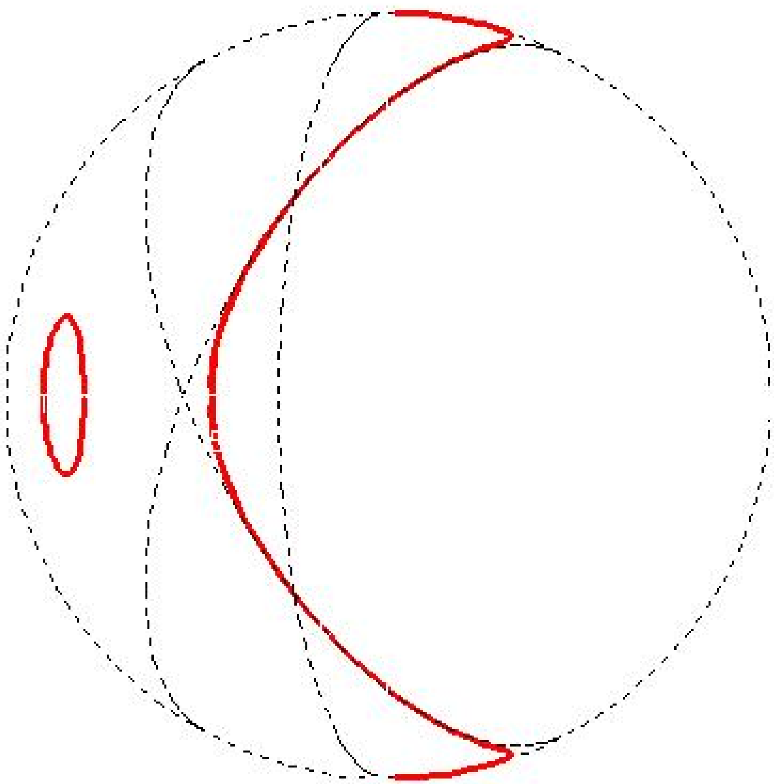
with a ball centered at the origin, one obtains the following 7 pictures for these Möbius types.



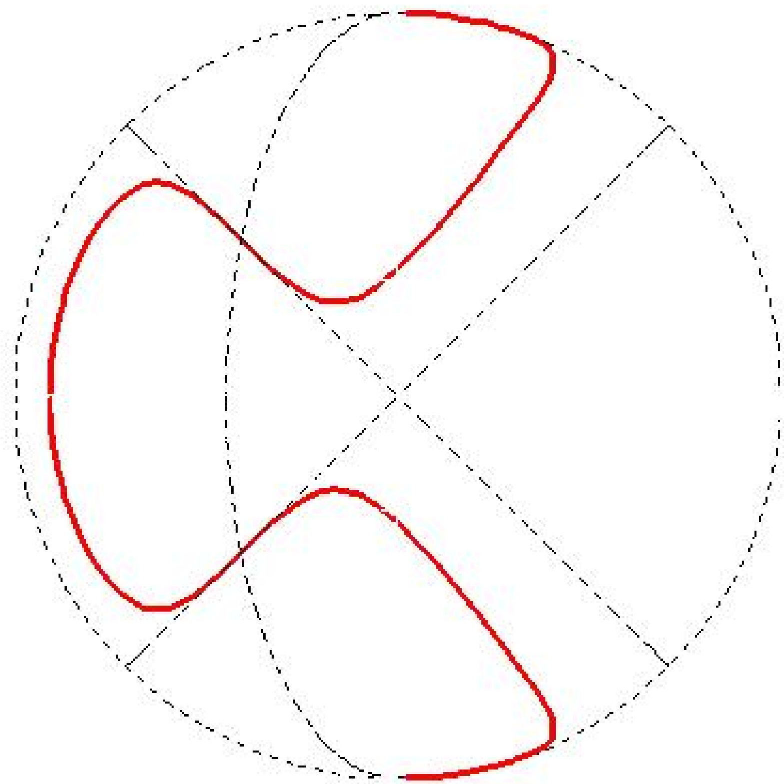
Gattung 1 (pura–a)



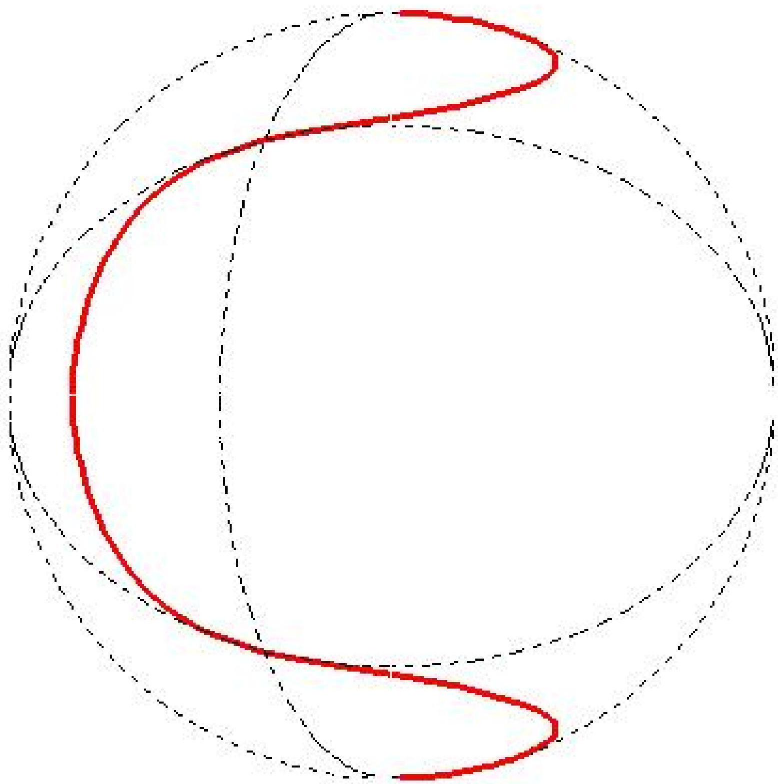
Gattung 2 (punctata)



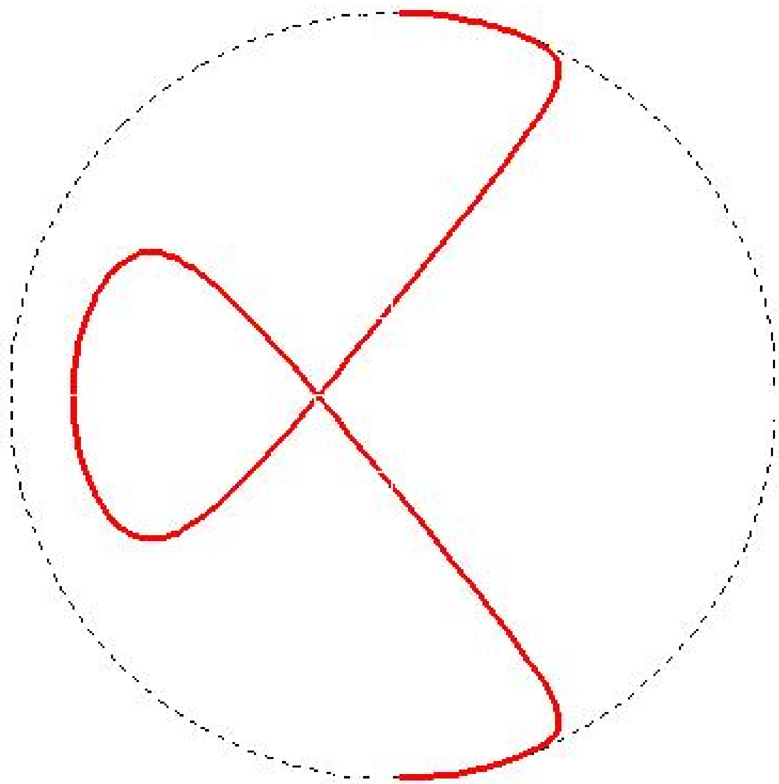
Gattung 3 (campanif. cum ovali)



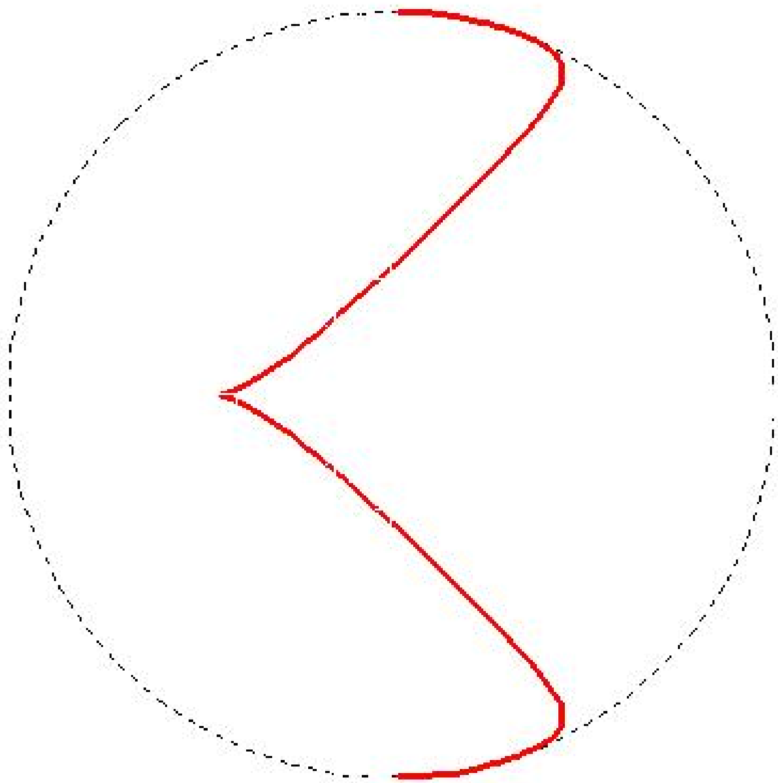
Gattung 4 (pura–c)



Gattung 5 (pura–b)



Gattung 6 (nodata)



Gattung 7 (cuspidata)

Gattung 7 (the Groningen IWI cuspidata)

Gattung 3 (the Groningen IWI campaniformis cum ovali)