# Exercises Computational Fluid Dynamics

## General remarks

During the course several computer exercises have to be made. These exercises are to be solved in 'pairs' of one or two students. A (short) written report of the findings has to be handed in the following week; discussion of the homework with other 'pairs' of CFD students is not allowed during that period.

All software required for the exercises is available by downloading the files from the Nestor pages for this course:

`Computational Fluid Dynamics > Course Information`

An alternative is to download them from

`http://www.math.rug.nl/~veldman/Colleges/CFD/CFD-Exercises.html`

## Exercise 1 – Artificial diffusion

**Files and commands** Required files: `cfd_1.m, cfd_upw.m, ns_exact.m`
The program is started in Matlab with the command `cfd_1`.

### Description

Consider the inhomogeneous convection-diffusion equation

$$\frac{\mathrm{d}\phi}{\mathrm{d}x} - k\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} = S \quad \text{on} \quad 0 \leq x \leq 1, \quad \text{with} \quad \phi(0) = 0, \quad \phi(1) = 1.$$

The right-hand side is given by

$$S(x) = \begin{cases} 2 - 5x & , \quad 0 \leq x \leq 0.4, \\ 0 & , \quad 0.4 < x \leq 1. \end{cases}$$

In this exercise an upwind discretization of the above equation is applied. The discrete solution for three values of the diffusion coefficient $k$ is plotted in one picture. Also the exact solution for one value of $k$ can be included in this picture.

### Questions

1. Firstly, study the discrete solution for a grid with $N = 20$ grid points, at the following values of $k$: $k = 0.1$, $0.01$ and $0.001$. These values correspond with mesh-Péclet numbers $P = 0.5$, $5$ and $50$, respectively. One would expect (or at least hope) that these solutions tend to the (analytical) solution for $k = 0$. Hence, compare the discrete solutions to the exact solution at $k = 0$ (ignore the Matlab warnings concerning division by zero). What do you observe; is there any such relation?

2. Next, compare the same three discrete solutions at $k = 0.1$, $0.01$ and $0.001$ (again for $N = 20$) to the exact solution at $k = 0.025$. Now, there seems to be some relation between the upwind limit for $k \to 0$ and the exact solution at $k = 0.025$. Explain this relation.

3. Finally, study once more the upwind solutions at $k = 0.1$, $0.01$ and $0.001$, but now on a grid with $N = 40$ grid points. Which exact solution, i.e. at which value of $k$, corresponds with the upwind limit for $k \to 0$?