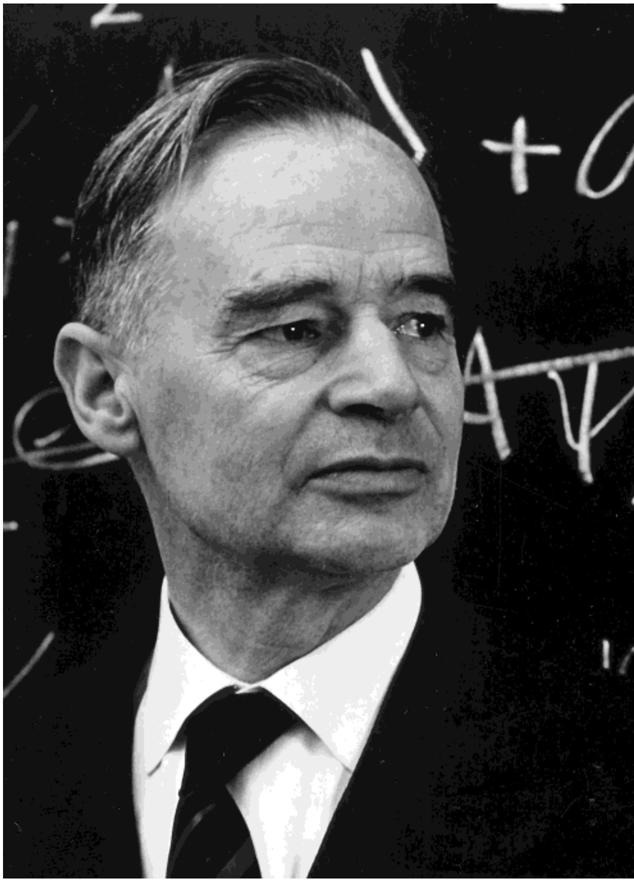

B. L. VAN DER WAERDEN, ca. 1965

Courtesy of the Mathematics Institute, Oberwolfach

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IN MEMORIAM

Bartel Leendert van der Waerden (1903–1996)

Bartel Leendert van der Waerden, one of the great mathematicians of this century, died in Zürich on January 12, 1996. Born in Amsterdam on February 2, 1903, he showed an interest in mathematics at an early age. His father, an engineer and a teacher of mathematics with an active interest in politics—to the left but not a Communist—wanted his son to play outside, away from the mathematics books indoors. The young van der Waerden, however, preferred playing the solitary game called "Pythagoras." This consisted of pieces which could be moved around freely and with which a square, a rectangle, or a triangle could be constructed in a variety of ways. Somewhat later, having somehow come up with the concept of the cosine, he rediscovered trigonometry, starting from the law of cosines. As a schoolboy, he regularly went to the reading hall of the public library in Amsterdam, where he studied a treatise in analytic geometry by Johan Antony Barrau, a professor at Groningen. Part II of that book contained many theorems insufficiently proven, even insufficiently formulated, and prompted van der Waerden to write to the author. Barrau's reaction? Should he leave Groningen, the university would have to nominate van der Waerden as his successor! This actually proved prophetic. Barrau moved to Utrecht in 1927, and in 1928, van der Waerden declined an offer from Rostock to accept his first chair at Groningen.

After van der Waerden finished school, he studied mathematics and physics from 1919 to 1924 at the University of Amsterdam under L. E. J. Brouwer; Roland Weitzenböck; Gerrit Mannoury; Hendrik de Vries; and the physicist Jan Dirk van der Waals, son of the Nobel laureate. The most famous of these was Brouwer, who, despite the fact that his most important research contributions were in topology, never gave courses in topology and who only lectured on the foundations of his intuitionism. (He apparently no longer felt convinced of his results in topology because they were not correct from the point of view of intuitionism.) Although van der Waerden studied invariant theory with Weitzenböck, he felt he learned the most from Mannoury, the mathematician who introduced topology to Holland.

In 1924, van der Waerden took his final examination with de Vries, whose course in classical algebra he had very much liked. It included subjects like determinants and linear equations, symmetric functions, resultants and discriminants, Sturm's theorem on real roots, Sylvester's "index of inertia" for real quadratic forms, and the solution of cubic and biquadratic equations by radicals. Van der Waerden supplemented this course by studying Galois theory and other subjects from Heinrich Weber's textbook on algebra. He also read Felix Klein's *Studien über das Ikosaeder* and thoroughly studied the theory of invariants. While he was still a

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university student, concepts like the "dimension" and the "generic point" of an algebraic variety, principles like Schubert's "principle of conservation of number," and theorems such as Max Noether's "fundamental theorem on algebraic functions" occupied his thoughts. In "On the Sources of My Book *Moderne Algebra,*" van der Waerden explained that

> invariants was a mighty tool in algebraic geometry. According to Felix Klein's "Erlanger Programm", every branch of geometry is concerned with those properties of geometrical objects that are invariant under a certain group. However, when I studied the fundamental papers of Max Noether, the "Father of Algebraic Geometry" and the father of Emmy Noether, and the work of the great Italian geometers, notably of Severi, I soon discovered that the real difficulties of algebraic geometry cannot be overcome by calculating invariants and covariants. [9, 32]

Since van der Waerden finished his studies in a very short time, his father agreed to support him for an additional year of study at Göttingen. At this time, David Hilbert, Gustave Herglotz, Edmund Landau, Carl Runge, Richard Courant, Emmy Noether, and Felix Bernstein comprised the permanent staff of the Mathematics Institute, while the corps of *Privatdozenten* included Alexander Ostrowski, Hellmuth Kneser, Paul Bernays, and historian of science Otto Neugebauer. Prominent guests—like Hermann Weyl, Constantin Carathéodory, John von Neumann, Carl Ludwig Siegel, Helmut Hasse, Richard Brauer, Heinz Hopf, Paul Alexandroff, Oswald Veblen, G. D. Birkhoff, and Norbert Wiener—came from Germany and the rest of the world. Provided with a letter of recommendation from Brouwer to Kneser, van der Waerden went to Göttingen in 1924. From Kneser, he learned more topology during the course of their walks together after lunch; other masters in topology were Alexandroff and Kazimierz Kuratowski. Van der Waerden also met Hilbert, whom he found affable and who invited him to his house; but, above all, he met Emmy Noether, who opened up a new world before him. From her, he learned that the tools with which his mathematical questions could be handled had already been developed, and she told him what to study: Ernst Steinitz's fundamental paper, "Algebraische Theorie der Körper"; F. S. Macaulay's Cambridge tract on polynomial ideals, *Algebraic Theory of Modular Systems*; the famous 1882 paper on algebraic functions, "Theorie der algebraischen Funktionen einer Veränderlichen," by Richard Dedekind and Heinrich Weber; and her own papers on "Idealtheorie in Ringbereichen" and "Eliminationstheorie und allgemeine Idealtheorie." Taking advantage of Göttingen's well-equipped mathematics library, van der Waerden started learning abstract algebra and working on his main problem, a determination of the proper foundations for algebraic geometry.

In 1925, van der Waerden returned to Holland for his mandatory year of military duty, which he served at the marine base in Den Helder. While there, he wrote his Ph.D. thesis, "De algebraiese grondslagen der meetkunde van het aantal," which presented a program for the foundation of algebraic geometry. Since a dissertation was only accepted in Holland at that time if it was written in Dutch or Latin, van der Waerden published his foundational work with its proofs in several papers in the *Mathematische Annalen,* while the Dutch dissertation itself consisted only of the statements of the theorems without proofs.

His year of military service over, van der Waerden went to Hamburg in 1926 to study with Emil Artin, Erich Hecke, and Otto Schreier thanks to a Rockefeller fellowship procured for him by Courant on the recommendation of Emmy Noether. Artin gave a course on algebra that summer, and, based on van der Waerden's lecture notes, the two planned to coauthor a book on algebra for Springer-Verlag's "Yellow Series." As van der Waerden worked out his notes and showed Artin one chapter after another, Artin was so satisfied that he said "Why don't you write the whole book?"

Van der Waerden returned to Göttingen in the summer semester of 1929 and met his future wife, Camilla Rellich, sister of the mathematician Franz Rellich. It was love at first sight, and that love lasted a lifetime. She created an ambiance in which he could create his mathematics. As Camilla van der Waerden remembers it, "We met in July and were married in September. Then we went to Groningen. And everything was well and beautiful, even very beautiful. After a while Emmy Noether called, I remember distinctly, and said, 'Time to end the honeymoon, back to work again!' Then he sat down and finished the book in one stretch" [1, 136; 2, 316]. This book, *Moderne Algebra I* (from the third edition onward it was called simply *Algebra*), was published in 1930 and was based on lectures by both Artin and Emmy Noether. It opened with explanations of such fundamental notions as group, normal divisor, factor group, ring, ideal, field, and polynomial and proceeded to proofs of theorems such as the *Homomorphiesatz* and the unique factorization theorems for integers and polynomials. Since these things were generally known, van der Waerden in most cases just reproduced Artin's proofs from his notes. In their review of part I, Hans Hahn and Olga Taussky explained the book's underlying philosophy:

> By algebra, van der Waerden means an algebra, often called an abstract algebra, in which the constants and indeterminates are not considered as real or complex numbers but as elements of some abstract set. Between these elements relations are given which have to satisfy the appropriate axioms. This interpretation, which has been common in group theory for a long time, is also accepted in point set theory and topology as well as in the theory of limits (Fréchet), and spread to all of algebra from a paper by Steinitz. This abstract interpretation provides deeper insight into the logical structure of the separate disciplines [and] an exact determination of the significance of the separate theorems, and [it] does not impede, although one might presume this at first, but rather facilitates understanding noticeably. All these advantages of the abstract method are abundant in van der Waerden's book. It is marked by complete rigor, great clarity, and easy comprehensibility. [4, 11–12]

Whereas Part I of *Moderne Algebra* principally treats the theories and problems from which algebra originated, Part II, published in 1931, deals with its subsequent development, a development to which van der Waerden contributed decisively [7, 3]. From the beginning, the book was an indispensible aid, and generations of mathematicians have learned and continue to learn algebra from it.

In 1931, van der Waerden went to Leipzig especially attracted by the physicists Werner Heisenberg and Friedrich Hund. In Göttingen, he had studied the methods of mathematical physics from Courant and his students Hans Lewy and Kurt Friedrichs. Heisenberg and Hund held a joint seminar in Leipzig, and there van der

Waerden learned physics. This resulted in a book on the methods of group theory in quantum mechanics, *Die gruppentheoretische Methode in der Quantenmechanik,* which was published in Berlin in 1932. Well received by the physics community, the book sold out rapidly and was later rewritten in English.

Also, in 1933, "Zur algebraischen Geometrie (ZAG) I, Gradbestimmung von Schnittmannigfaltigkeiten einer beliebigen Mannigfaltigkeit mit Hyperflächen" was published in the *Mathematische Annalen.* With this series (ZAG I–XX) of articles, van der Waerden began an algebrization of Italian algebraic geometry. ZAG IX was a joint work with Wei-Liang Chow and contains one of Chow's most influential results: To each projective variety a homogeneous polynomial is associated in such a way that the association extends to a homomorphism from the additive monoid of effective cycles in projective space to the multiplicative monoid of homogeneous polynomials; this association is compatible with the Zariski topology (in other words, if one cycle is a specialization of another, then the associated Chow form is also a specialization). In the introduction to ZAG IX, the proof of this result is attributed to Chow. Van der Waerden went on to develop a global intersection theory in ZAG XIV, from which followed further investigations on intersection multiplicities by Pierre Samuel, Claude Chevalley, and Jean-Pierre Serre. Completely new points of view were introduced into the area by André Weil, the first to formulate the notion of a *local* algebraic theory of intersections; by Oskar Zariski; by Chow; and by the Grothendieck school, principally Pierre Deligne [8].

The van der Waerden family remained in Leipzig until 1945, then they fled from the incessant bombardments to the outskirts of Graz, Austria, where Camilla van der Waerden's mother lived. When the Americans arrived in July 1945, the van der Waerdens were considered "displaced persons" and were taken back to Holland by bus. The situation there proved difficult, as the Dutch resented them for having remained in Nazi Germany. (Van der Waerden's personal file, kept in the archives of the University of Leipzig, shows, however, that he spoke out in favor of young Jews. The University honored him in 1984 with an honorary doctorate.) Following his return, van der Waerden had no financial resources until Hans Freudenthal helped him to secure a position at Shell, the Royal Dutch Petroleum Company. Working together with Shell engineers, van der Waerden solved optimization problems, a topic that he came very much to enjoy.

The year 1947 found him in the United States at The Johns Hopkins University in Baltimore. When Hopkins made him a permanent offer, he declined, accepting instead a chair at Amsterdam City University and suggesting that Hopkins extend the offer to Chow. Van der Waerden remained in Amsterdam for only 2 years; in 1951, he moved to Zürich, where he stayed for the rest of his life.

As early as 1938, van der Waerden's first paper on the history of science, "Die Entstehungsgeschichte der ägyptischen Bruchrechnung," appeared in *Quellen und Studien zur Geschichte der Mathematik* [3, 243; 10, 183]. As a student in Amsterdam, he had already become interested in the history of mathematics through a course given by Hendrik de Vries. Following this, he read Euclid and some of Archimedes, and, during his first sojourn in Göttingen, he attended Neugebauer's lectures as well as his course on Greek mathematics. Neugebauer, however, worked principally on Egyptian mathematics, his thesis having been on this topic, and van der Waerden found that research especially stimulating. When van der Waerden later visited Neugebauer in Copenhagen, he became captivated by the latter's accounts of Babylonian astronomy. This new interest deepened even further during his vacations near Graz as a result of conversations with the assyriologist Ernst Weidner. Finally, in Leipzig, van der Waerden's good friend and colleague, the philosopher Hans-Georg Gadamer, again stimulated his prior interest in Greek mathematics. All this resulted in the book, *Science Awakening,* which was first published in Dutch in 1950. Translated into many languages, most recently into Persian in 1994, van der Waerden's account met the needs of mathematicians desirous of a sense of the origins of their subject.

The book received very favorable reviews. For example, Dirk Struik pointed out that "this is the first book which bases a full discussion of Greek mathematics on a solid discussion of pre-Greek mathematics" [6]. Struik specifically mentioned those aspects of the text which he considered new: (1) an analysis of Freudenthal's hypothesis which held that the Hindus, after having developed the decimal system, learned the sexagesimal system and the zero through Greek astronomy and thus acquired their own position system; (2) arguments showing that Thales actually gave a logical proof of the four theorems ascribed to him by Eudemos; (3) a reconstruction of Pythagorean number theory based on the arithmetical books of Euclid's *Elements*; (4) a comparison of the geometrical algebra of the *Elements* with Babylonian texts showing that the Greeks, and especially the Pythagoreans, took their material from the Babylonian tradition; (5) an explanation of how Theodoros of Kyrene actually showed the irrationality of what we call $\sqrt{2}, \sqrt{3}, \ldots, \sqrt{17}$, and why he stopped at $\sqrt{17}$; (6) a demonstration of certain deficiencies in Archytas's logical thinking by means of Book VIII of the *Elements*; (7) an explanation of the mathematics in Plato's posthumous *Epinomis,* based on the work of Archytas; and (8) an analysis of Book X of the *Elements,* which is ascribed to Theaitetos.

Many more publications in this field followed. Eventually, van der Waerden's interests shifted to the history of astronomy, Sanskrit astronomy, Aryabhata, and Persian astronomy. His last paper, "The Motion of Venus in Greek, Egyptian and Indian Texts," appeared in *Centaurus* in 1988. In many cases, as in the medieval Indian method for computing planetary positions, van der Waerden used his own mathematical insight to elucidate the works of the ancient and medieval mathematicians and astronomers [5, 152–157]. His last book, *A History of Algebra from al-Khwārizmī to Emmy Noether,* appeared in 1985 and provided a personal account of the development of algebra. Starting with al-Khwārizmī, from whose treatise the word "algebra" is derived, van der Waerden traced the development through the ages to modern times. In his view, modern algebra began with Galois, who first investigated the structure of fields and groups and showed that these two structures are closely connected. After Galois, the efforts of the leading algebraists were mainly directed toward the investigation of the structure of rings, fields, algebras, and the like. Van der Waerden tracked such investigations up through roughly the middle of the 20th century in a discussion that comprises some two-thirds of the book and that includes some of his own contributions. Only van der Waerden could have given us this fascinating account.

Upon his retirement from the University of Zürich, van der Waerden was honored with the formation of an Institute for the History of Mathematics, to which he generously donated a substantial portion of his private library. When I visited him in the summer of 1994, he invited me into his library and told me to take what I wanted. These books are not only a valuable addition to my own library but, above all, are a very cherished souvenir of an extremely kind and generous man. I last visited van der Waerden in February 1995. He was physically weak but mentally very alert and in good spirits. As always, he was an attentive and inspiring listener, and as always, I enjoyed being with him and his wife in the warmth of their home. I feel fortunate to have known him.

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REFERENCES

1. Yvonne Dold-Samplonius, Bartel Leendert van der Waerden befragt von Yvonne Dold-Samplonius, *NTM* (*International Journal of History and Ethics of Natural Sciences, Technology and Medicine*), New Series **2** (1994), 129–147.
2. Yvonne Dold-Samplonius, Interview with Bartel Leendert van der Waerden, trans. Yvonne Dold-Samplonius and Hugo Rossi, *Notices of the American Mathematical Society* **44** (1997), 313–320.
3. Günther Eisenreich, B. L. van der Waerden's Wirken von 1931 bis 1945 in Leipzig, in *100 Jahre Mathematisches Seminar der Karl-Marx-Universität, Leipzig*, Berlin: VEB Deutscher Verlag der Wissenschaften, 1981, pp. 218–244.
4. Hans Hahn and Olga Taussky, Review of van der Waerden's *Moderne Algebra,* Part I, *Monatshefte für Mathematik und Physik* **39** (1932), Literaturberichte 11–12, reprint ed., New York: Johnson Reprint Corporation, 1958.
5. Jan P. Hogendijk, B. L. van der Waerden's Detective Work in Ancient and Medieval Mathematical Astronomy, in *Nieuw Archief voor Wiskunde,* 4th series **12** (1994), 145–158.
6. Dirk J. Struik, Review of van der Waerden's *Ontwakende Wetenschap: Egyptische, Babylonische en Griekse Wiskunde, Mathematical Reviews* **12** (1951), 381.
7. Olga Taussky, Review of van der Waerden's *Moderne Algebra,* Part II, *Monatshefte für Mathematik und Physik* **40** (1933), Literaturberichte 3–4, reprint ed., New York: Johnson Reprint Corporation, 1958.
8. Bartel L. van der Waerden, The Foundation of Algebraic Geometry from Severi to André Weil, *Archive for History of Exact Sciences* **7** (1971), 171–180.
9. Bartel L. van der Waerden, On the Sources of My Book *Moderne Algebra, Historia Mathematica* **2** (1975), 31–40.
10. Bartel L. van der Waerden, Bibliography of B. L. van der Waerden, ed. Jaap Top and Lynne Walling, *Nieuw Archief voor Wiskunde,* 4th series **12** (1994), 179–193.

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