High dimensional covering thresholds for beta polytopes

RTG 2131: Introduction of Gilles Bonnet

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Bochum, 22nd January 2017

Birth Bergerac

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Studies	Bordeaux, Manchester, Barcelona

Interests: Geometry

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Teaching	Paris

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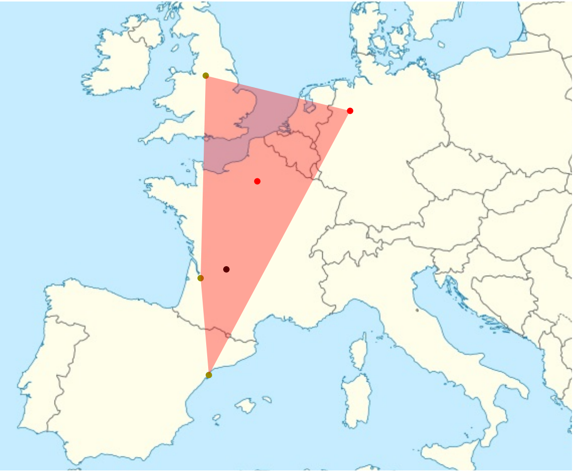

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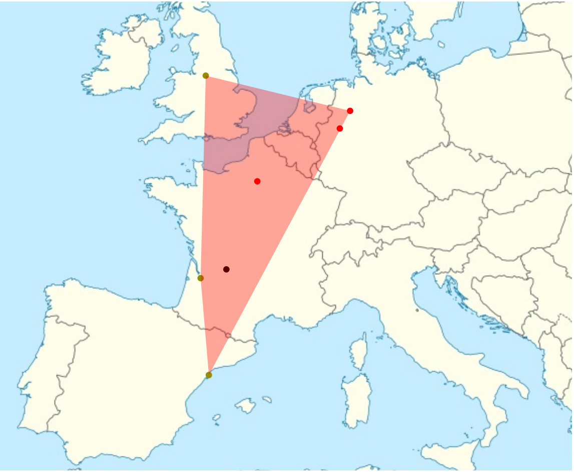
Teaching PhD	Paris Osnabrueck

Research Interests: Geometry, Polytopal approximation, Stochastic & integral geometry, Random tessellations, Random polytopes, Poisson point processes



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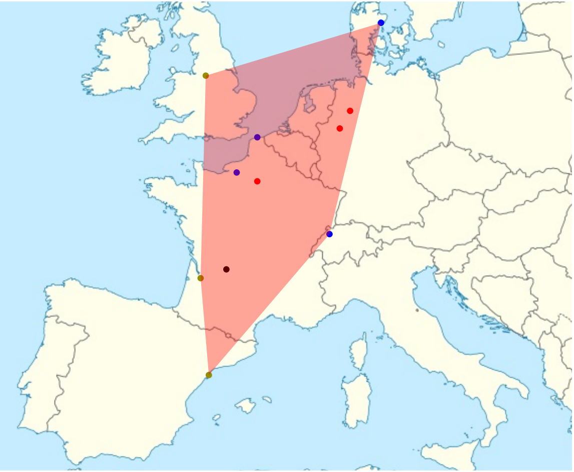

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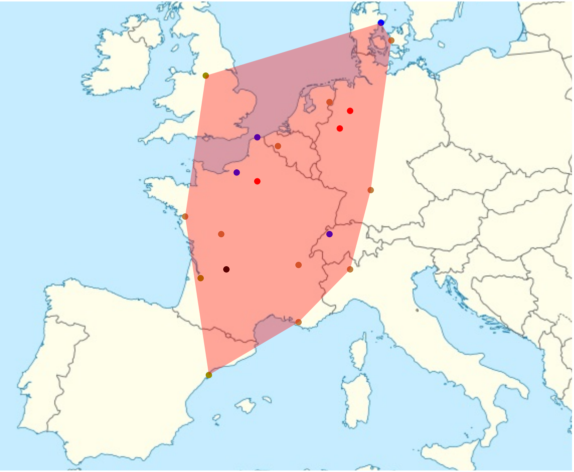

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Conferences	Sandbjerg Estate Ghiffa-Oggebbio Poitiers Lille Marseille ...

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Theorem: Threshold for random polytopes in the unit ball
B., G. Chasapis, J. Grote, D. Temesvari, N. Turchi (2018)

Let $N = N(n) > n$ and $X_1, \dots, X_N$ be i.i.d. random uniform vectors in B^n. Let also $\varepsilon \in (0,1)$. Let $B_{N,n} = \mathrm{conv}\{X_1, \dots, X_N\}$. Then

$$\lim_{n\to\infty} \frac{\mathbb{E}\,\mathrm{Vol}_n(B_{N,n})}{\mathrm{Vol}_n(B^n)} = \begin{cases} 0 & \text{if } N \leq \exp\left((1-\varepsilon)(\frac{n+1}{2})\log n\right) \\ 1 & \text{if } N \geq \exp\left((1+\varepsilon)(\frac{n+1}{2})\log n\right). \end{cases}$$

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- Generalisation to the Beta model:

$$\text{density } \simeq (1-\|x\|^2)^\beta,\ \beta > -1,\ x \in B^n.$$

- Analogous statements for the Beta prime model:

$$\text{density } \simeq \left(1 + \frac{\|x\|^2}{\sigma^2}\right)^{\beta},\ \beta > \frac{n}{2},\ \sigma > 0,\ x \in \mathbb{R}^n.$$

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$\beta = 0 \Rightarrow$ uniform distribution on the ball B^n
$\beta \to -1 \Rightarrow$ uniform distribution on the sphere S^{n-1} [P. Pivovarov '07]

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$\sigma^2 = 2\beta \to \infty \Rightarrow$ normal distribution in $\mathbb{R}^n$ [P. Pivovarov '07]

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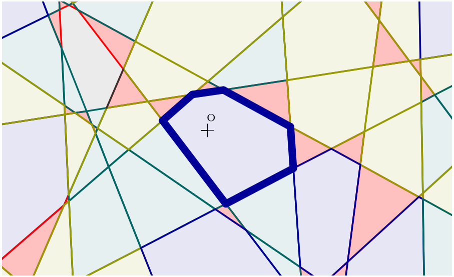

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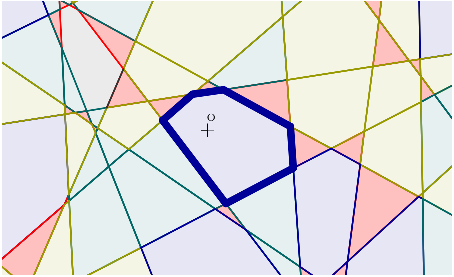

THANK YOU!