| | $H_{typ}$ | $\mathbb{E}f_{d-1}(P_{n,d})$ |
|---|---|---|
| $1 \ll n-d \ll \sqrt{d}$<br>$n-d \sim \rho\sqrt{d}$ | $dH_{typ} - \rho\sqrt{2/\pi} \xrightarrow{D} Z$ | $\binom{n}{d} 2^{-n+d+1} e^{\frac{(n-d)^2}{\pi d} + O(\frac{(n-d)^3}{d^2}) + o(1)}$ * |
| $\sqrt{d} \ll n-d \ll d$ | $\frac{d^{3/2}}{n-d} H_{typ} \xrightarrow{P} \frac{2}{\pi}$ | |
| $n-d \sim \rho d$ | $\sqrt{d} H_{typ} \xrightarrow{P} r_\rho$ | $e^{dc_\rho + o(d)}$ |
| $\ln n \ll d \ll n$ | $\sqrt{\frac{d}{\ln(n/d)}} H_{typ} \xrightarrow{P} \sqrt{2}$ | $\left[(4\pi + o(1)) \ln\left(\frac{n}{d}\right)\right]^{\frac{d-1}{2}}$ * |
| $\ln n \sim \rho d$ | $\sqrt{1 - H_{typ}^2} \xrightarrow{P} e^{-\rho}$ | $\left[2\pi(e^{2\rho} - 1)d(1 + o(1)\right]^{\frac{d-1}{2}}$ |
| $\ln n \gg \rho d$ | $-\frac{d-1}{\ln n} \ln(1 - H_{typ}^2) \xrightarrow{P} 2$ | $nK_d\sqrt{1 - \left(\frac{d^3}{n^2}\right)^{\frac{1}{d-1}}}(1 + o(1))$ |
| $\ln n \gg d \ln d$ | $\frac{n}{2\sqrt{\pi d}}(1 - H_{typ}^2)^{\frac{d}{2}} - \sqrt{d} \xrightarrow{tv} Z$ | $nK_d(1 + o(1))$ * |
| $d$ fixed | $nc_d(1 - H_{typ}^2)^{\frac{d-1}{2}} \xrightarrow{tv} \Gamma_{d-1}$ | |

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