

WEAK CONVERGENCE OF THE INTERSECTION POINT PROCESS OF POISSON HYPERPLANES

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η_t (stationary, isotropic) Poisson hyperplane process of intensity $t, t > 0$.

$\eta_{t,R}$ restriction of η_t to the set of hyperplane intersecting the ball of radius $R, R > 0$.

$\Xi_{t,R} = \{H_1 \cap \dots \cap H_d : (H_i)_i \in \eta_{t,R}^{(d)}\}$ intersection point process.

ζ Poisson point process on $\mathbb{R}^d \setminus \{0\}$ whose intensity measure M has density $x \mapsto c_2 \|x\|^{-(d+1)}$.

ASSUMPTIONS

$$R = t^{-\frac{d}{d+1}},$$

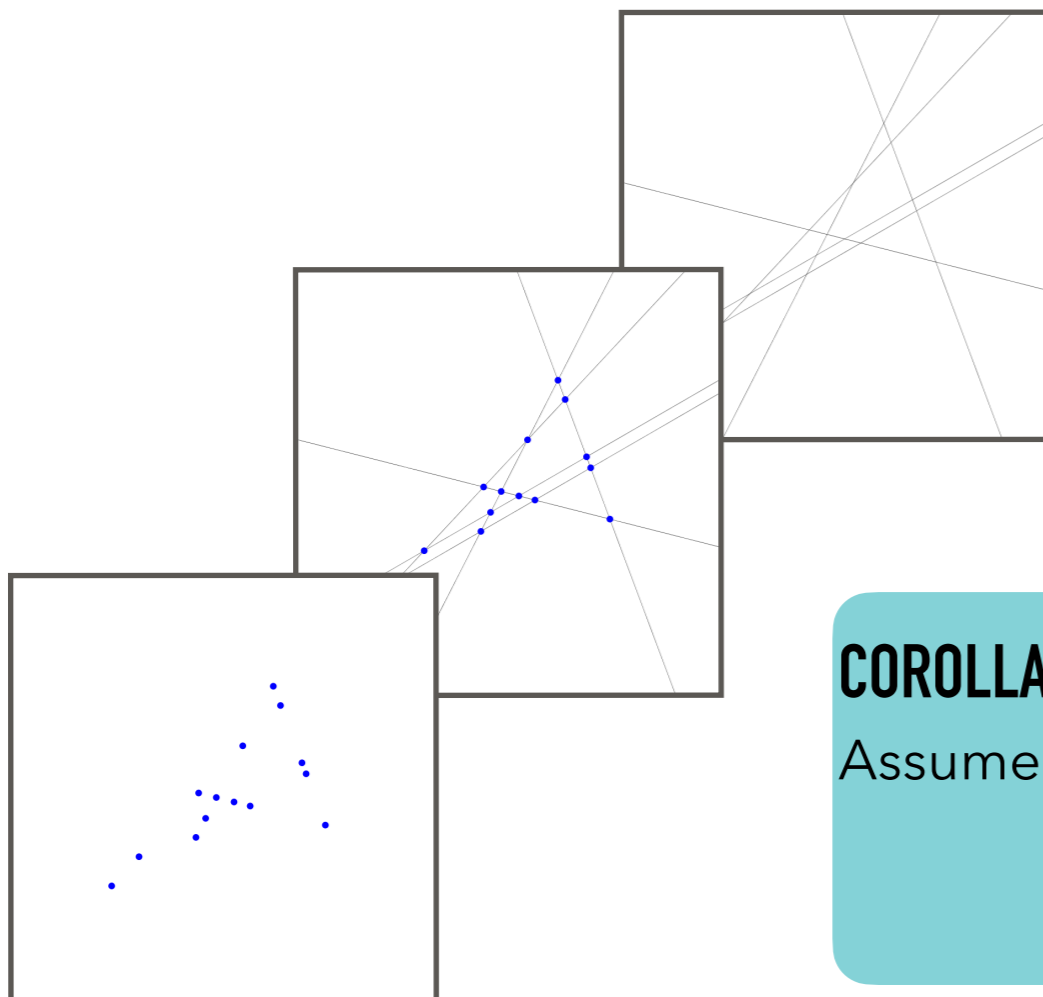
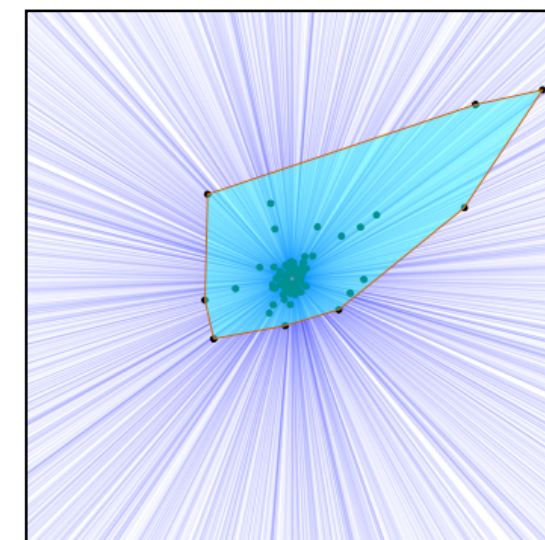
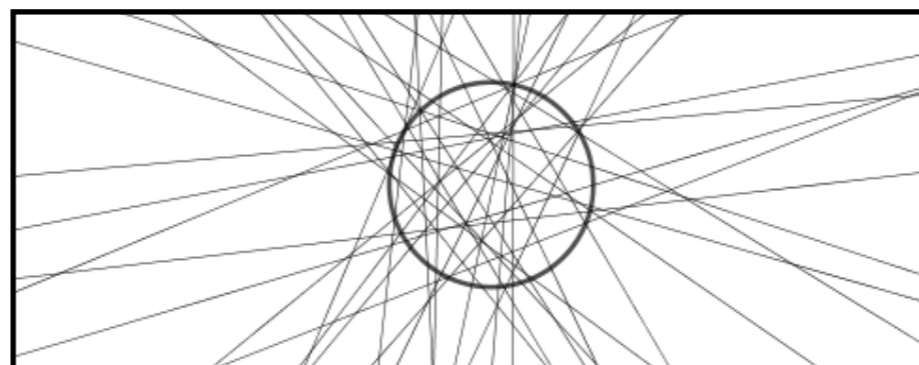
$$t \rightarrow \infty.$$

THEOREM (Bound on the K-R distance)

$$d_{\text{KR}}(\Xi_{t,R}|_{(B_r)^c}, \zeta|_{(B_r)^c}) \leq ct^{-\frac{1}{d+1}} \ln(t)r^{-3}.$$

THEOREM (Convergence in distribution)

$$\Xi_{t,R} \xrightarrow{d} \zeta, \quad t \rightarrow \infty.$$



COROLLARY (Convergence of the convex hull)

$$\text{conv } \Xi_{t,R} \xrightarrow{d} \text{conv } \zeta, \quad t \rightarrow \infty.$$

and, for any $k \in \{0, \dots, d\}$,

$$f_k(\text{conv } \Xi_{t,R}) \xrightarrow{d} f_k(\text{conv } \zeta), \quad t \rightarrow \infty.$$

COROLLARY (Disproof of a conjecture from [Devroye, Toussaint, 1993])

Assume that $d = 2$. Then

$$\liminf_{t \rightarrow \infty} \mathbb{E} f_0(\text{conv } \Xi_{t,R}) \geq \mathbb{E} f_0(\text{conv } \zeta) = \frac{\pi^2}{2} > 4.$$