High dimensional random polytopes

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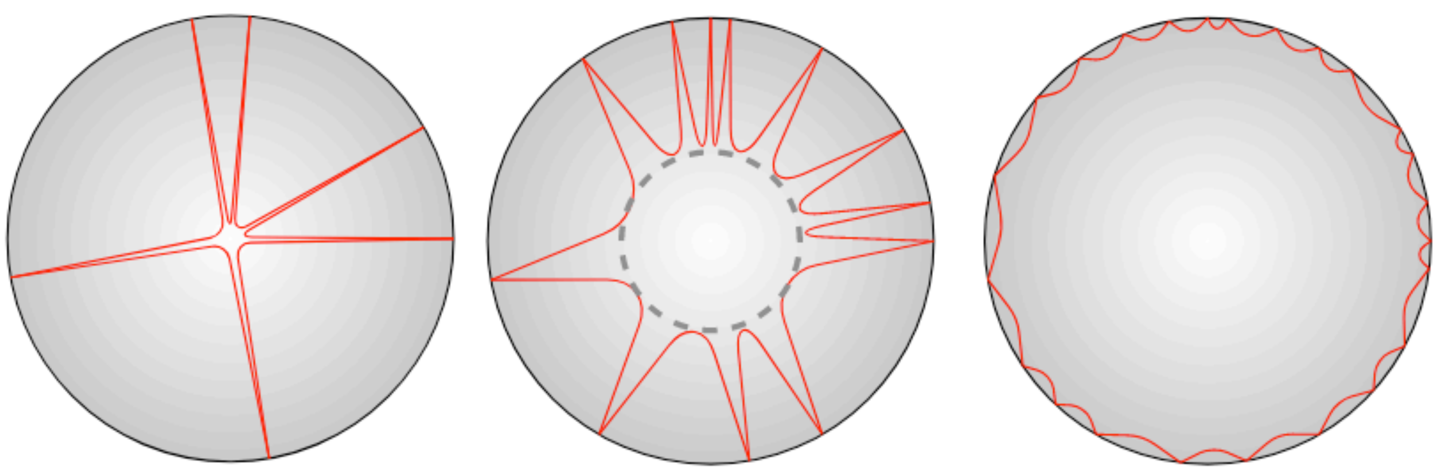
joint works with Chasapis, Grote, Kabluchko, O'Reilly, Temesvari, Turchi

$P_{n,d} = \mathrm{conv}(X_1, \dots, X_n)$ with $X_1, \dots, X_n \in \mathbb{S}^{d-1}$ i.i.d uniformly distributed.



(a) **sub-exponential regime** $(\ln n)/d \to 0$. Facets' heights $\simeq O(1/\sqrt{d})$.

(b) **exponential regime** $(\ln n)/d \to \alpha$. Facets' heights $\simeq \sqrt{1-e^{-\alpha}}$.

(c) **super-exponential regime** $(\ln n)/d \to \infty$. Facets' heights $\simeq 1$.

Precise results depending on the regime $d = d(n)$.
For example:
CLT for H_{typ} if $n - d = O(\sqrt{d})$ or $\ln n \gg d \ln d$.

$$\int_{h_1}^{h_2} c_1 (1-h)^{\frac{d^2-2d-1}{2}} \left(c_2 \int_{-1}^{h} (1-s^2)^{\frac{d-3}{2}} \, \mathrm{d}s \right)^{n-d} \mathrm{d}h.$$

Volume Threshold:

$$\lim_{d\to\infty} \frac{\mathbb{E}\, V_d(P_{n,d})}{V_d(B^d)} = \begin{cases} 0 & \text{if } n \le d^{(1-\varepsilon)(\frac{d}{2})}, \\ 1 & \text{if } n \ge d^{(1+\varepsilon)(\frac{d}{2})}. \end{cases}$$

Volume Phase Transition:

If $x \in \mathbb{R}$ is fixed and $n = \left(\frac{d}{2x+o(1)}\right)^{\frac{d}{2}}$, then

$$\lim_{d\to\infty} \frac{\mathbb{E}\, V_d(P_{n,d})}{V_d(B^d)} = e^{-x}$$

Also: extension of the *threshold* and *phase transition* to *beta distributions* and *intrinsic volumes*