



rijksuniversiteit
 groningen



Buffon's needle problem

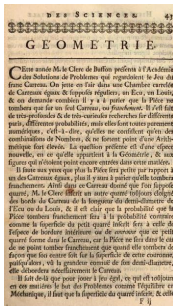
A journey from an old recreational problem to modern mathematics

Gilles Bonnet (University of Groningen)

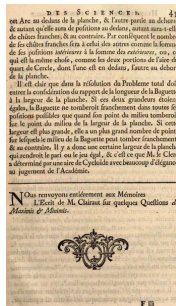
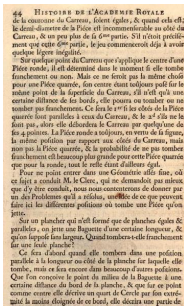
Open days mathematics, 12 Nov 2021



(a) Georges-Louis Leclerc, Comte de Buffon, 1707-1788

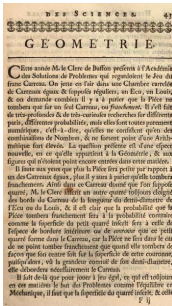


(b) de l'Acad. Roy. des. Sciences (1733), 43–45

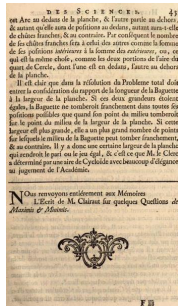
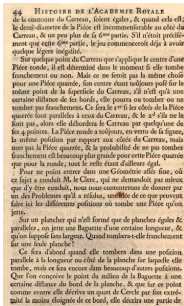




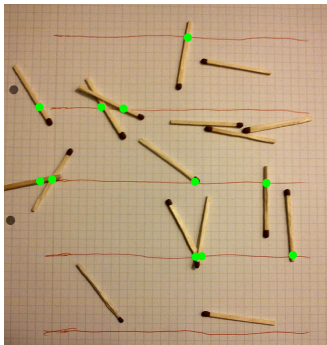
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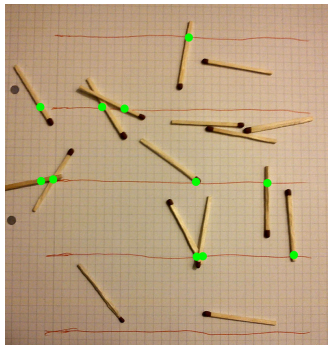
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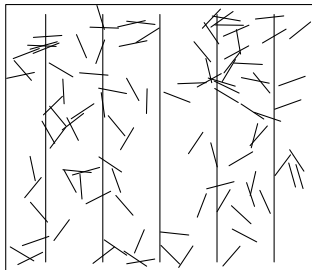
Suppose we have a floor made of parallel strips of wood, each the same width, and we drop a needle onto the floor. What is the probability that the needle will lie across a line between two strips?



(a) Buffon's needle match problem

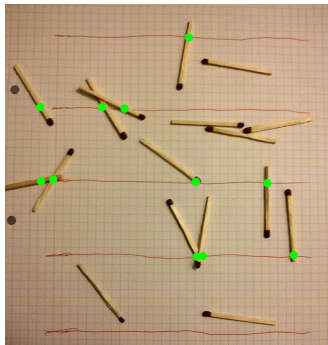


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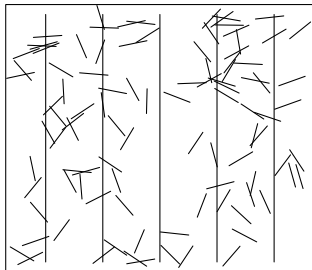


31 out of 100 needles are crossing
Empirical probability = 31 %

(b) Computer simulations



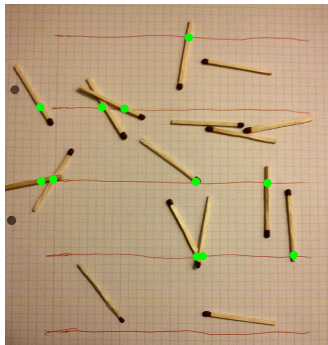
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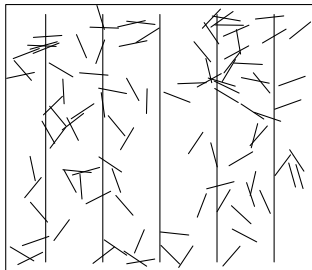
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- Is 31% a good estimation of the *real* probability?



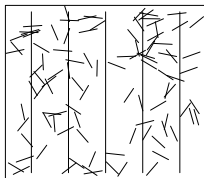
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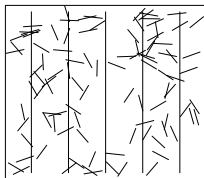
(b) Computer simulations

- Is 31% a good estimation of the *real* probability?
- If I throw again 100 needles, will I see again 31% of needles crossing a line?



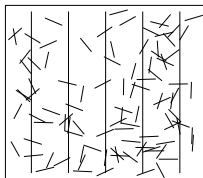
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(a) Is it 31% ?



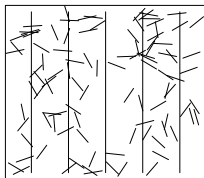
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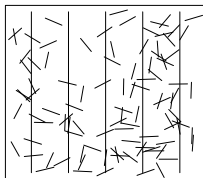
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(b) 33%?



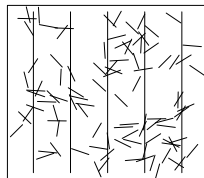
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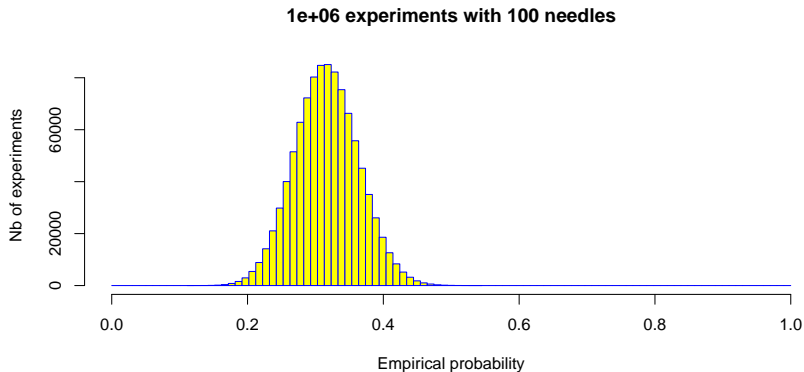
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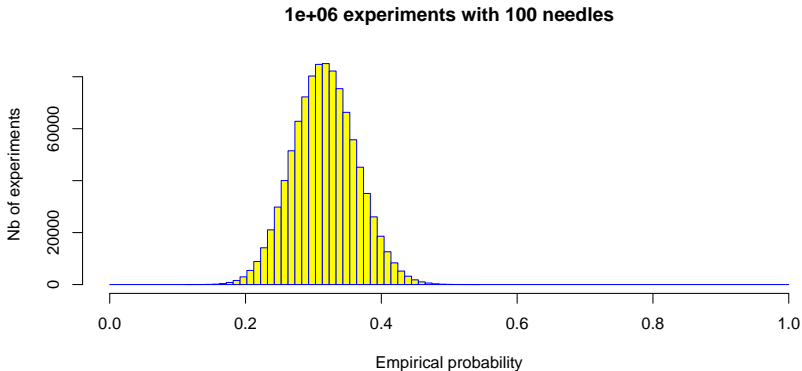
(b) 33%?



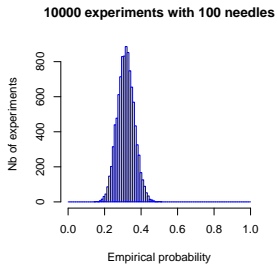
36 out of 100 needles are crossing
Empirical probability = 36 %

(c) or 36%?



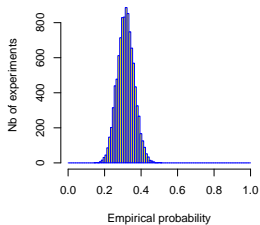


- We need to throw more needles!

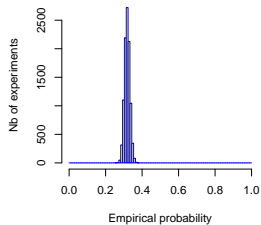




10000 experiments with 100 needles

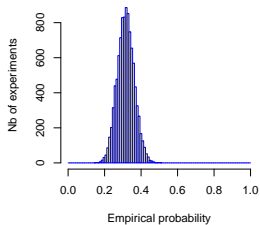


10000 experiments with 1000 needles

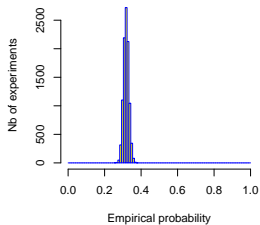




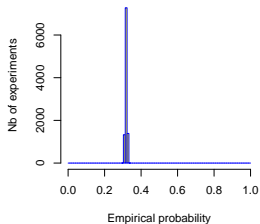
10000 experiments with 100 needles



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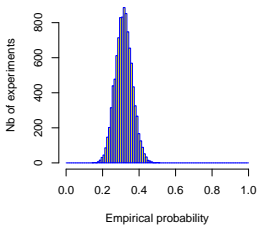


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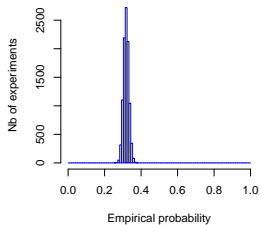




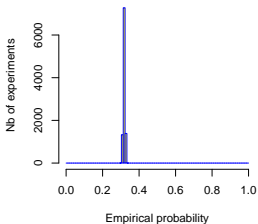
10000 experiments with 100 needles



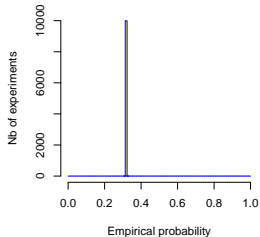
10000 experiments with 1000 needles



10000 experiments with 10000 needles



10000 experiments with 1e+05 needles





Empirical probability of crossing = 0.318



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$$\frac{1}{\text{Empirical probability of crossing}} = \frac{1}{0.318} = 3.14\dots$$

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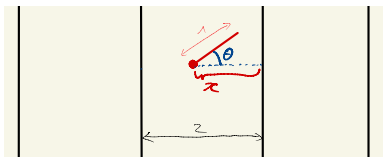


?

- x = distance between the most left point of a the needle and the next vertical line,
- θ = angle between the needle and a horizontal line.

Two observations:

1. $x \in [0, 2]$ and $\theta \in [-\pi/2, \pi/2]$ are *uniformly* distributed
2. The needle crosses a line precisely when $x \leq \cos \theta$.

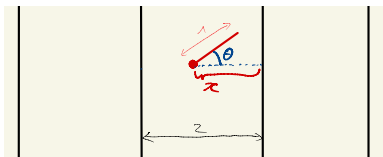


$$\text{Probability} = \int_{-\pi/2}^{\pi/2} \int_0^2 \mathbf{1}(x \leq \cos \theta) \frac{dx}{2} \frac{d\theta}{\pi}$$

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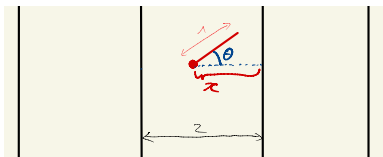


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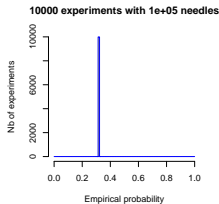


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Doing our simulations we have observed.

1. When we throw a looooooot of needles the proportion of needles crossing a line is always close to the probability of crossing.

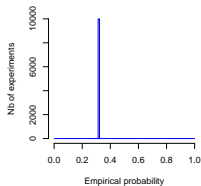




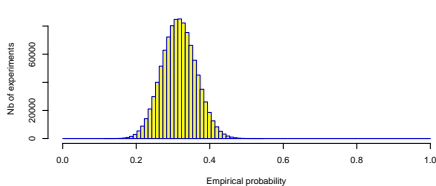
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2. When we do many experiments with many needles the histogram has the shape of a bell 🛎

10000 experiments with $1e+05$ needles



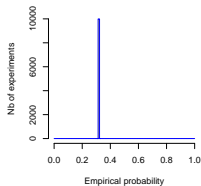
$1e+06$ experiments with 100 needles



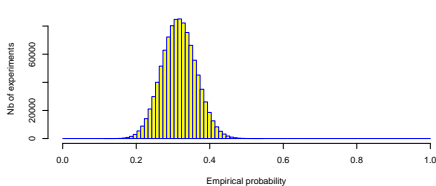
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$1e+06$ experiments with 100 needles



Go study mathematics!! and learn about

1. The law of large numbers.
2. The central limit theorem.



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Thank you for your attention!