rijksuniversiteit
groningen

Random tessellations

Gilles Bonnet

CogniGron & Bernoulli Institute for Mathematics, University of Groningen

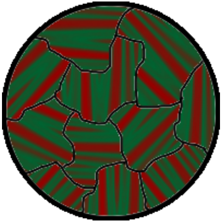
CogniGron@Work session, Groningen, June 27 2022

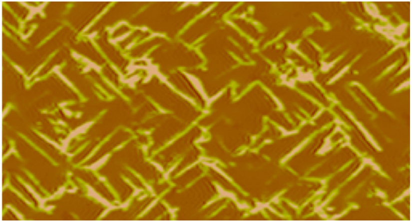
- Gilles Bonnet
- French
- Lived in: France, UK, Spain, Germany, Netherlands
- Since September 2021:
  - Ass. Prof. in the Probability Group
  - and member of the CogniGron

(a) Escher

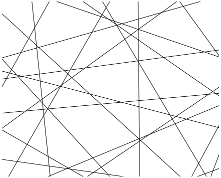
(b) Mud crack

Tessellation for some of the CogniGron members: Polarity / Domain walls?





Tessellations for me



(a) Line tessellation

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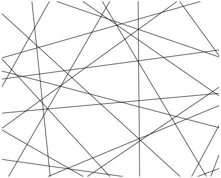

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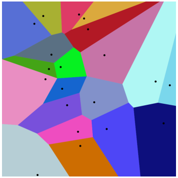

(b) Voronoi tessellation

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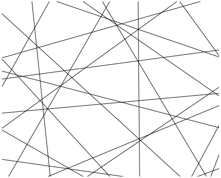

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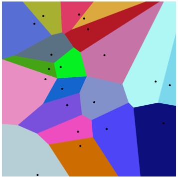

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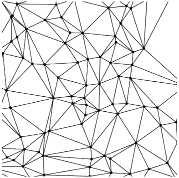
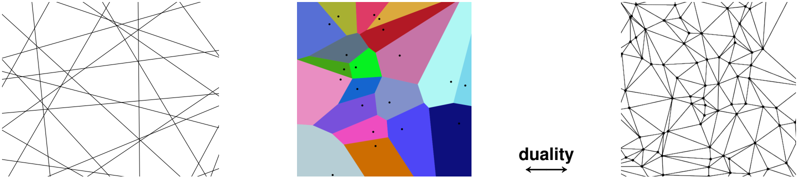
duality ⟷



(c) Delaunay tessellation

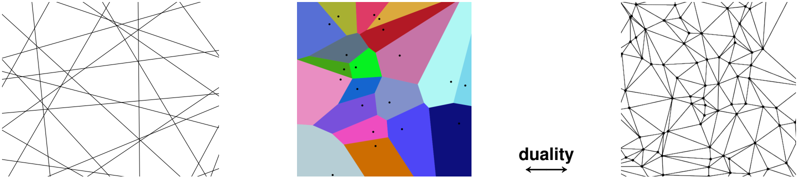


(a) Line tessellation (b) Voronoi tessellation (c) Delaunay tessellation

Two graphs associated to a tessellation:

1. Skeleton of the tessellation:
   - Vertices = Vertices of the cells in the tessellation,
   - Edges = Edges of the cells in the tessellation.

Tessellations for me



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Two graphs associated to a tessellation:

1. Skeleton of the tessellation:
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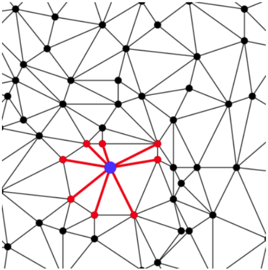
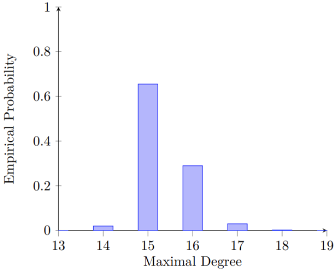
   - Edges = Edges of the cells in the tessellation.
2. Contact graph:
   - Vertices = Center of the cells in the tessellation,
   - Edges = Edges between cells touching each other.

One example of things I have done



(a) Vertex of maximal degree



(b) In a window $[0, 1\,000]^2$

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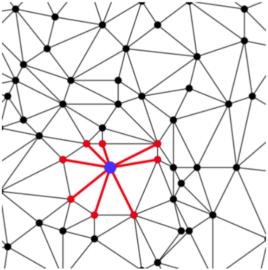

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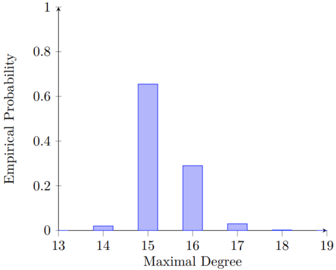

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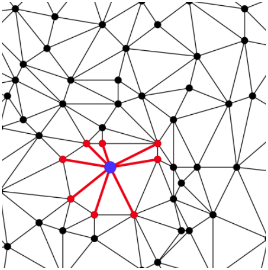
$\Delta_\rho :=$ maximal degree in a planar Poisson-Delaunay graph over all nodes in $\left[0, \rho^{\frac{1}{2}}\right]^2$.



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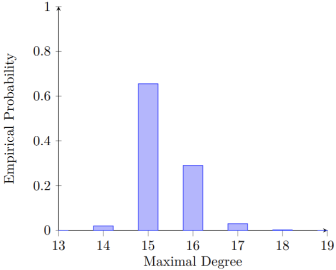

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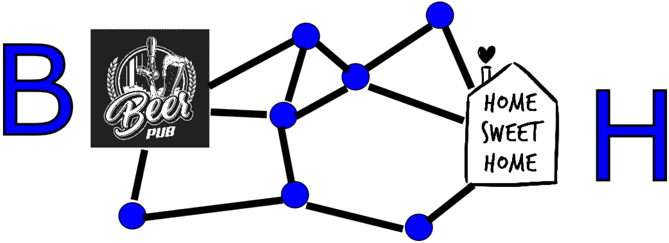
Theorem **[G.B., N. Chenavier; Bernouilli 2020]**

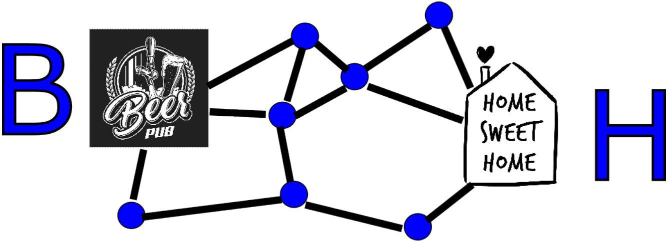
There exists a deterministic function $\rho \mapsto I_\rho \in \mathbb{N}$, such that

1. $\mathbb{P}(\Delta_\rho \in \{I_\rho, I_\rho + 1\}) \to 1$;
2. $I_\rho \sim \frac{1}{2} \frac{\log \rho}{\log \log \rho}$.



Let $(W_n)_{n\in\mathbb{N}}$ be a random walk on a graph, starting at B.

What is the probability that the drunkard reaches home without returning to the pub?



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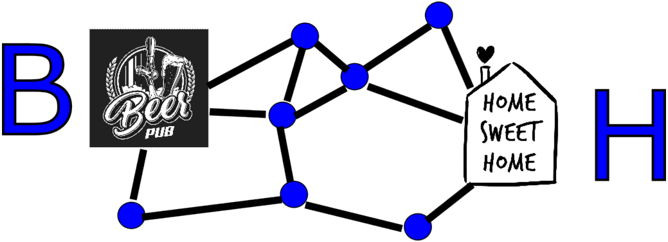
Consider this graph as an electrical network where each edge has resistance 1.



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Theorem

$$\mathbb{P}(\text{The drunkard returns home}) = \frac{1}{\text{degree}(B) \times \text{Resistance}(B, H)}.$$

Ideas of things I could work on in the CogniGron

1. Random tessellations to model some material. What question are of interest for people in the CogniGron?

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4. ...

THANK YOU!