



rijksuniversiteit
 groningen



Random tessellations

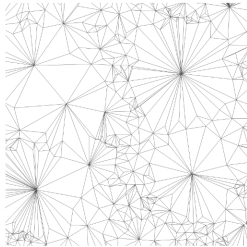
Gilles Bonnet

CogniGron & Bernoulli Institute for Mathematics, University of Groningen

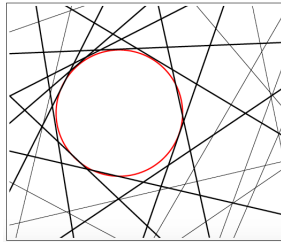
CogniGron@Work session, Groningen, June 27-2022 October 30 2023



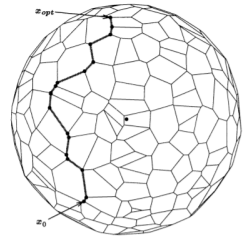
- Gilles Bonnet
- French
- Lived in: France, UK, Spain, Germany, Netherlands
- Since September 2021:
 - Ass. Prof. in the [Probability Group](#)
 - and member of the [CogniGron](#)



(a) What is the maximal vertex degree in a Beta-prime Delaunay triangulation?



(b) What is the volume of large empty regions in a Poisson hyperplane tessellation?



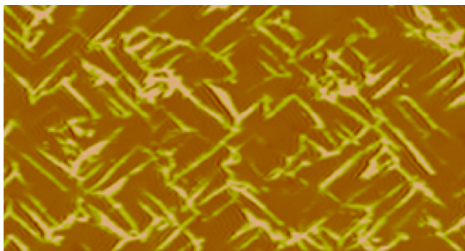
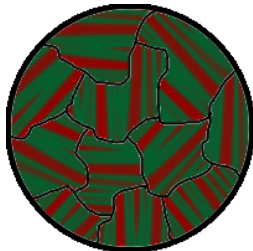
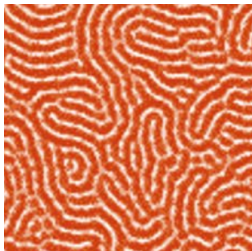
(c) What is the shortest path between vertices of a polytope defined by random linear constraints?

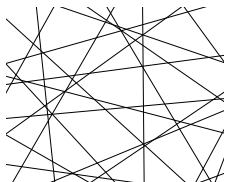


(a) Escher

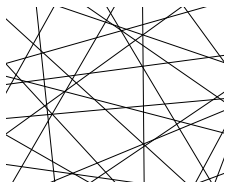


(b) Mud crack

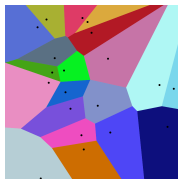




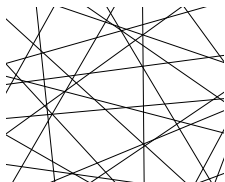
(a) Line tessellation



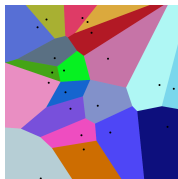
(a) Line tessellation



(b) Voronoi tessellation

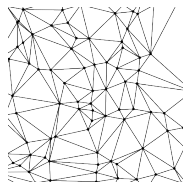


(a) Line tessellation

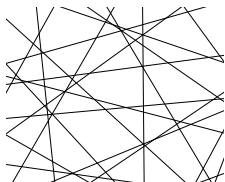


(b) Voronoi tessellation

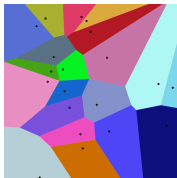
duality
↔



(c) Delaunay tessellation

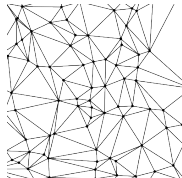


(a) Line tessellation



(b) Voronoi tessellation

duality
↔

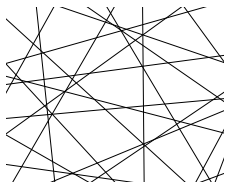


(c) Delaunay tessellation

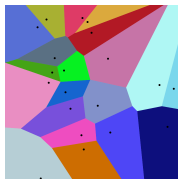
Two graphs associated to a tessellation:

1. Skeleton of the tessellation:

- Vertices = Vertices of the cells in the tessellation,
- Edges = Edges of the cells in the tessellation.

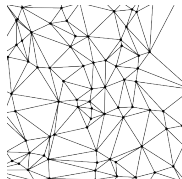


(a) Line tessellation



(b) Voronoi tessellation

duality
↔



(c) Delaunay tessellation

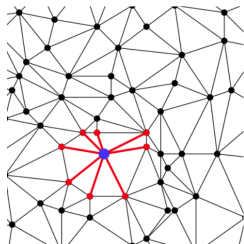
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1. Skeleton of the tessellation:

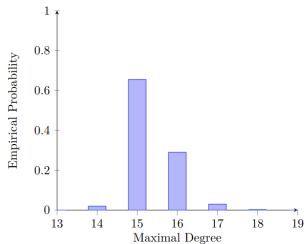
- Vertices = Vertices of the cells in the tessellation,
- Edges = Edges of the cells in the tessellation.

2. Contact graph:

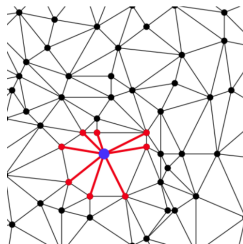
- Vertices = Center of the cells in the tessellation,
- Edges = Edges between cells touching each other.



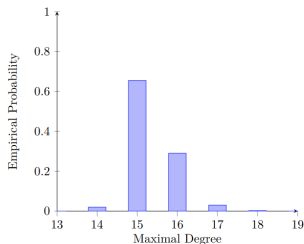
(a) Vertex of maximal degree



(b) In a window $[0, 1000]^2$

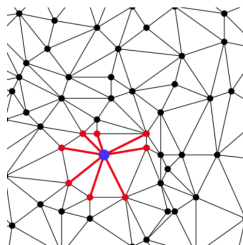


(a) Vertex of maximal degree

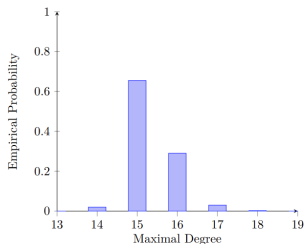


(b) In a window $[0, 1000]^2$

$\Delta_\rho :=$ maximal degree in a planar Poisson-Delaunay graph over all nodes in $\left[0, \rho^{\frac{1}{2}}\right]^2$.



(a) Vertex of maximal degree



(b) In a window $[0, 1000]^2$

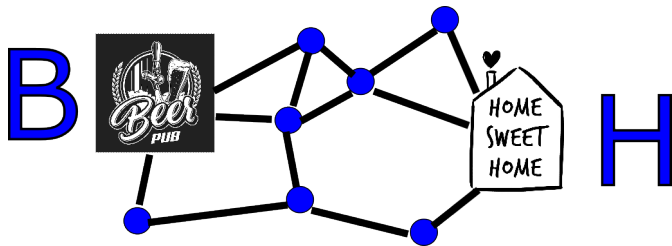
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Theorem

[G.B., N. Chenavier; Bernoulli 2020]

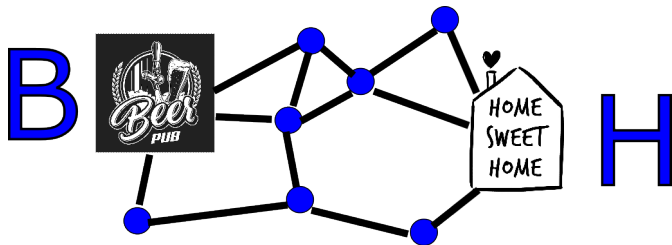
There exists a deterministic function $\rho \mapsto I_\rho \in \mathbb{N}$, such that

1. $\mathbb{P}(\Delta_\rho \in \{I_\rho, I_\rho + 1\}) \rightarrow 1$;
2. $I_\rho \sim \frac{1}{2} \frac{\log \rho}{\log \log \rho}$.



Let $(W_n)_{n \in \mathbb{N}}$ be a random walk on a graph, starting at B .

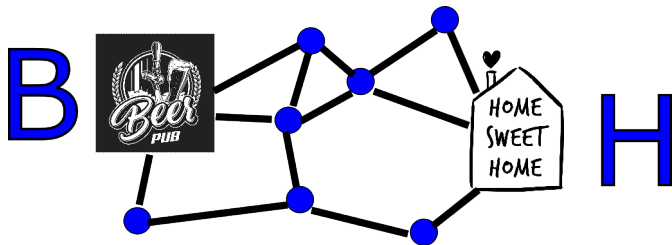
What is the probability that the drunkard reaches home without returning to the pub?



Let $(W_n)_{n \in \mathbb{N}}$ be a random walk on a graph, starting at B .

What is the probability that the drunkard reaches home without returning to the pub?

Consider this graph as an electrical network where each edge has resistance 1.



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What is the probability that the drunkard reaches home without returning to the pub?

Consider this graph as an electrical network where each edge has resistance 1.

Theorem

$$\mathbb{P}(\text{The drunkard returns home}) = \frac{1}{\text{degree}(B) \times \text{Resistance}(B, H)}.$$



1. Random tessellations to model some material. What question are of interest for people in the CogniGron?



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2. Random walk and network of memristors?



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2. Random walk and network of memristors?
3. Random graph as a base for a neural network?
4. ...

THANK YOU!