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‘Which model ...?’ is the wrong question

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Statistics: making *decisions* in the presence of *uncertainty* (analysis)

and with limited *resources* (design)

Model as a conduit:

If I knew *the* model, then the analysis/inference would be efficient

— *absolutely not* true, and sometimes not relevant

Select a model and use it for all related inferences (*a bad idea*) *vs.*

Combine estimators for a particular purpose (e.g., min MSE for a target)

The false rationale for model selection:

Let's find a model that looks 'good', and then ...

— such a model is a *random* entity — *model uncertainty*

Examples in which the search for a model is/would be a distraction:

- Textbook ANOVA (one-way, with homoscedasticity and normality)
- Clinical trials for comparing two treatments (randomisation)
- Small-area estimation (inference about districts of a country)

What is 'good' inference (estimation, hypothesis test, confidence interval)?

Integrity: Adhere to *this* criterion, without any conditioning

A bad (*circular*) criterion: 'Good' means: based on a well selected model

One-way ANOVA

(Longford, 2005, JRSS A; 2008, SORT):

Textbook:

Test the hypothesis of equal means — use the selected-model estimator

This estimator is extremely inefficient in some common settings

Use the same model for estimating σ^2

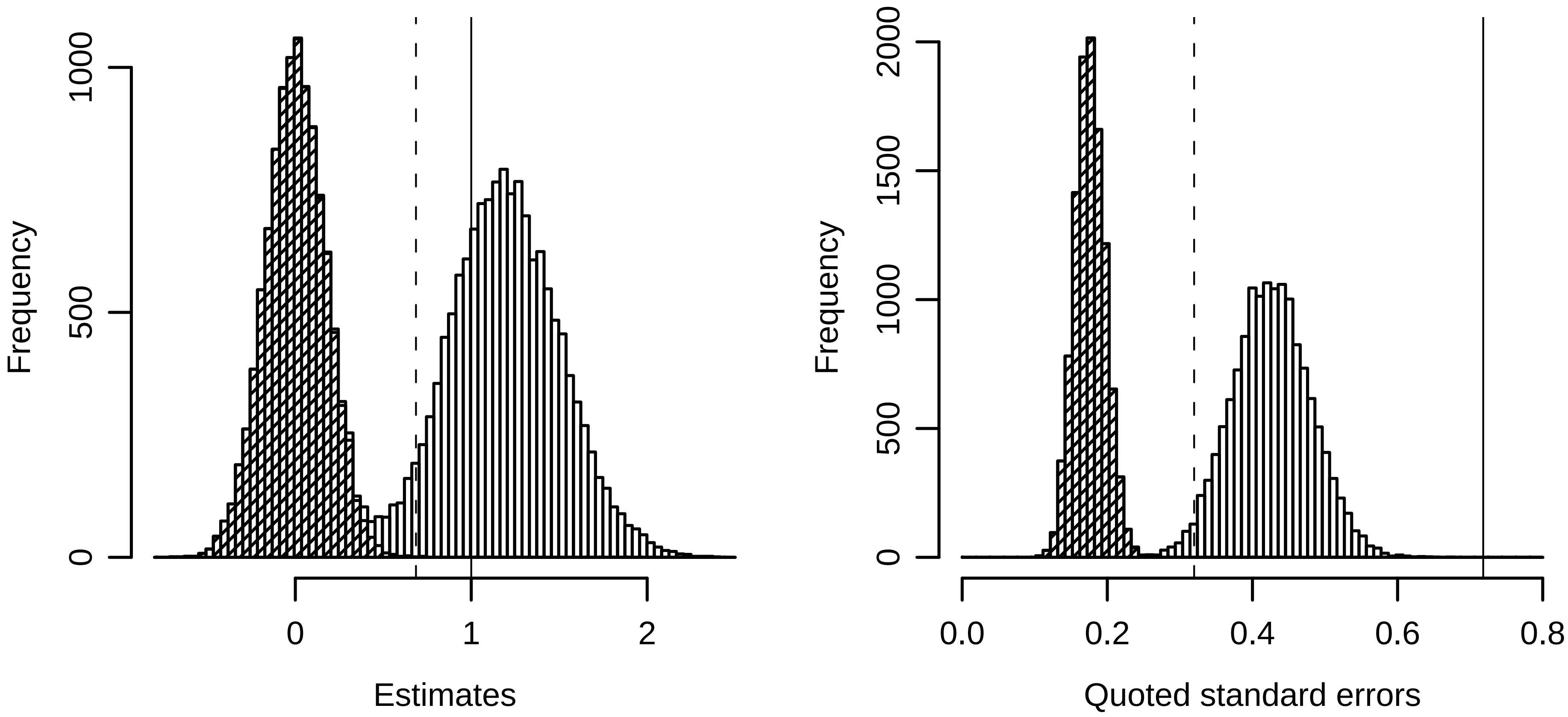
— a poor strategy (look at the degrees of freedom)

The problem is not with hypothesis testing, but *with model choice* in general

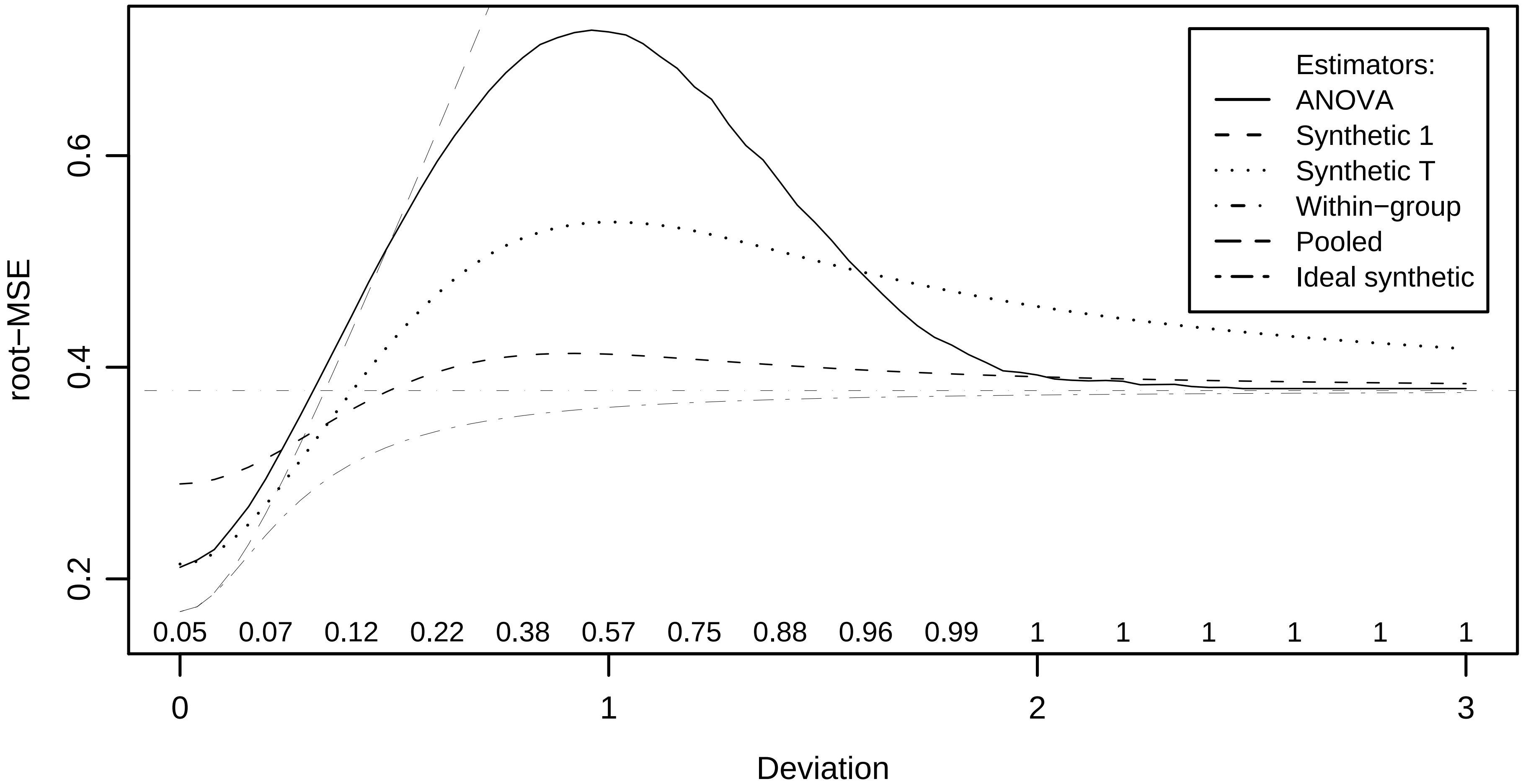
Bayes factors — no relief/no solution

Combine estimators, with weights specific to the target/estimand

The goal: small MSE



Gross inefficiency of the selected-model based estimator in one-way ANOVA (Longford, 2008; SORT)



Root-MSE of alternative estimators as functions of the deviation $\mu_1 - \mu$

(Longford, 2008; SORT)

Model selection

Elementary estimators: $\hat{\mu}_1 \sim \mathcal{N}\left(\mu_1\,, \frac{1}{n_1}\,\sigma^2\right)$ and $\hat{\mu} \sim \mathcal{N}\left(\mu, \frac{1}{n}\,\sigma^2\right)$

Model selection: $\mathcal{I}$ — indicator of selecting model A

$$
\begin{aligned}
\hat{\mu}_1^{\dagger} &= (1-\mathcal{I})\hat{\mu}_1 + \mathcal{I}\hat{\mu} \\
\mathrm{E}\left(\hat{\mu}_1^{\dagger}\right) &= \mu_1 + p_{\mathrm{B}}\left\{\mathrm{E}\left(\hat{\mu}\,|\,\mathcal{I}=1\right) - \mathrm{E}\left(\hat{\mu}_1\,|\,\mathcal{I}=1\right)\right\} \\
\mathrm{MSE}\left(\hat{\mu}_1^{\dagger}\,;\mu_1\right) &= p_{\mathrm{A}}\,\mathrm{var}\left(\hat{\mu}_1\,|\,\mathcal{I}=0\right) + p_{\mathrm{B}}\,\mathrm{var}\left(\hat{\mu}\,|\,\mathcal{I}=1\right) \\
&\quad + p_{\mathrm{A}}\left\{\mathrm{E}\left(\hat{\mu}_1\,|\,\mathcal{I}=0\right) - \mu_1\right\}^2 + p_{\mathrm{B}}\left\{\mathrm{E}\left(\hat{\mu}\,|\,\mathcal{I}=1\right) - \mu_1\right\}^2
\end{aligned}
$$

$p_{\mathrm{A}} = \mathrm{P}(\mathcal{I}=0)$; $p_{\mathrm{B}} = 1 - p_{\mathrm{A}}$. Note: $\mathcal{I}$ and $\hat{\mu}_1$ are correlated

Bias and *large* MSE are (almost) guaranteed

Combination of estimators

$$\tilde{\mu}_1 = (1 - b_1)\hat{\mu}_1 + b_1 \hat{\mu}\,,$$

$$\mathrm{MSE}\left(\tilde{\mu}_1\,;\mu_1\,|\,b_1\right) = b_1^2 \left\{ g_1\,\sigma^2 + (\mu_1 - \mu)^2 \right\} - 2b_1\,g_1\,\sigma^2 + \frac{\sigma^2}{n_1}$$

$$b_1^* = \frac{g_1}{g_1 + \dfrac{(\mu_1 - \mu)^2}{\sigma^2}}$$

where $g_1 = \frac{1}{n_1} - \frac{1}{n}$

Substitute $\hat{b}_1^*$ for b_1^*

Assess the consequences of over/under-estimating $(\mu_1 - \mu)^2/\sigma^2$

Scope for incorporating prior information, not necessarily Bayesian

(Longford, 2008, Chapter 1)

Clinical trials

Randomised allocation of subjects to two treatments

Estimation of the (constant) treatment effect

Including 'important' covariates in (a regression) analysis

— wasting degrees of freedom (Better model — Worse inference)

Crossover trials (within-subject contrasts) with design 'AB and BA'

Freeman (1989, *Stat. Med.*): Do not estimate the *carryover*

— waste of the data from the 2nd period

Longford (2001, *Stat. Med.*): Composition:

— reduce the 'weight' given to the 2nd period

Do not choose! — combine!!

Small-area estimation

A country with districts $d = 1, \ldots, D$ and quantities θ_d; 'national' value θ

Notation: $\hat{\theta}_d \sim \mathcal{Z}(\theta_d, v_d)$, $\hat{\theta} \sim \mathcal{Z}(\theta, v)$ and $c_d = \operatorname{cov}\left(\hat{\theta}_d, \hat{\theta}\right)$

A setting similar to ANOVA, except that $D \gg$ — random effects (??)

Sample size sufficient for estimating θ, but not for θ_d for some d

ANOVA irrelevant — composition of (unbiased) estimators $\hat{\theta}_d$ and $\hat{\theta}$:

$$\tilde{\theta}_d = (1 - b_d)\,\hat{\theta}_d + b_d\,\hat{\theta}$$

$$b_d^* = \frac{v_d - c_d}{v_d + v - 2c_d + \sigma_{\mathrm{B}}^2} \doteq \frac{v_d}{v_d + \sigma_{\mathrm{B}}^2}$$

$\sigma_{\mathrm{B}}^2 = \operatorname{var}_{\mathcal{D}}(\theta_d)$ — estimate σ_{B}^2 and study sensitivity $(\hat{v}_d)$

Extensions for auxiliary information (Longford, 2005)

Conclusion

The importance of model selection is vastly over-rated
because of not appreciating the pervasiveness of uncertainty
and ignoring the basics of conditional probabilities and distributions

Asymptotic theory (for AIC, BIC, u&IC) is questionable
for an essentially small-sample problem

Hypothesis testing (and intermediate decision, incl. model selection)
— a *steam engine* in the age of the *iPod*
because it is oblivious to the *consequences* of the errors I and II

Examples: 1. The Albanian long jumper Shenki Xhadni (2044);
2. Crossing the road in uptown Bendery during a Euro game.

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