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THE ROLE OF A STRUCTURAL MODE SHAPE BASED INTERACTION LAW TO SUPPRESS ADDED MASS INSTABILITIES IN PARTITIONED STRONGLY COUPLED ELASTIC STRUCTURE-FLUID SYSTEMS

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ABSTRACT

The physical level of interaction between fluid and structure can be either one-way or two-way depending on the direction of information exchange at the interface of fluid and solid. The former can be solved by a partitioned approach and weak coupling. In problems involving two-way fluid-structure interaction, using a partitioned approach and strong coupling, sometimes stability restriction is encountered. This is an artificial added mass effect, which is independent of the numerical time step. Unfortunately an accurate and efficient method to deal with all the different levels of interaction is scarce. Conventionally, relaxation is applied to remedy this problem. The computational cost is directly related to number of sub-iterations between fluid and structural solver at each time step. In this study, the source of this instability is investigated. A discrete representation of a basic added mass operator is given and instability conditions are assessed. A new method is proposed to relax this restriction, the idea essentially is to remove the instability source from the structure and move it to the fluid and solve it monolithically with the fluid. We call this an interaction law. An estimate of the structural response is derived from structural mode shapes. As a test case, a 2D dam break problem interacting with an elastic vertical flexible beam is selected. The interaction of fluid with the beam undergoes several stages. The breaking waves on the beam can increase the added mass drastically, therefore the added mass ratio increases as well. In such a cases, the asset of interaction law is better elaborated, while the stability condition requires very high relaxation without interaction law, but the relaxation can be lowered by only using first five beam mode shapes. As a consequence, the number of sub-iterations reduces by one order. The numerical observations confirm the reduction in computational time due to utilization of the interaction law.

NOMENCLATURE

FESI Fluid-Elastic-Structure Interaction

IL Interaction Law

INTRODUCTION

Fluid-Structure interaction is of a very interest in offshore hydrodynamic applications. These coupled system can be solved in a monolithic or partition manner. The former requires employing a method to solve both at the same time [1]. Besides complexity, current well developed structural and fluid packages do not suit to use in this way. On the other hand, partitioned approach brings a vast source of existing packages which can be coupled to each other. Within this coupling scheme, sometimes instabilities are observed. Part of this instabilities are related to the time step and they vanish as a small enough time step is applied [2]. This will increase computational time for a certain problem. Second source of instability is inherently in the system. In context of fluid-solid interaction, this is called artificial added mass effect instability. For compressible flows reducing time step can remove the destabilization source, but for incompressible flows the source of instability is time step independent [3]. To remedy the later, under relaxation need to be applied to the rate of information exchange at the interface of fluid and solid [4]. This two effects drastically slow down the solution of a partitioned coupled system.

Since the partitioned method has a lot of benefits as mentioned, lots of work is done in order to first identify the origin of instabilities and then speed up the whole solution by removing them from the system.

It is found that the ratio of the added mass to the structural mass is directly related to amount of under relaxation needed to be applied, therefore some researchers adopted methods to adjust relaxation factor with time. Some researchers [2] studied the effect of fluid and structure time integrator and also the predictor at the interface on the limit of stability.

In this study, further investigation is done to even speed up the solution and increase it's efficiency. The idea is to design a new predictor which can also implicitly anticipate structural response when the fluid is considered. For this purpose, according to [5] an approximate model of the structure needs to be selected and translated into fluid which will not increase level of complexity of the problem. Using the modal representation of the structure, different levels of linear simplification of the structure can be constructed employing part of the structural mode shapes. The structural mode shapes and natural frequencies needs to be provided in advance. This simplified structural interaction model acts as an boundary condition at the interface of fluid and structure.

It is observed that this new method can widen stability limit and therefore higher relaxations can be applied. As a consequence, the number of sub-iterations decreases drastically. This leads to the speed up of the entire solution.

In this paper, in the first part the equations of motion of the system and their boundary and initial conditions are described. Fluid solver in this study is the free surface flow finite volume solver COMFLOW [6] developed in University of Groningen and TUDelft and the structural solver is an finite element Euler-Bernoulli beam solver. In the next section, the implicit time marching method which is applied to couple this system is discussed. Time integrators and the predictor at the interface of fluid and structure which have a high importance for coupling scheme are elaborated. After that, in the section of stability, the stability of this system is investigated. The added mass operator is derived for a simplified from of fluid and structure equations. The added mass operator is then derived by a different method using modal representation of the structure. The idea of the interaction law is shown by comparing these two operators. Exploiting the later form of added mass operator, the asses of the interaction law is elaborated. In the last part, numerical test case is presented. The performance of pre-introduced interaction law is investigated and the results are discussed.

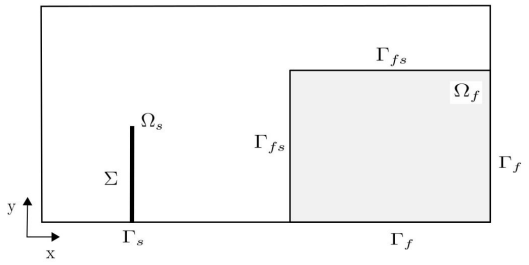

FIGURE 1. SKETCH OF FLUID AND STRUCTURE DOMAIN.

FLUID-ELASTIC-STRUCTURE PROBLEM STATEMENT

For brevity, the governing equations of the system is represented in 2 dimensional domain $\Omega \subset \mathbb{R}^2$. The FESI domain Ω as illustrated in Fig.1 consists of fluid domain Ω_f with boundary Γ_f and free surface boundary Γ_{fs} , while elastic-structure domain Ω_s has the boundary Γ_s and the boundary at the interface of the fluid and structure is Σ .

Fluid Domain

For the fluid the incompressible free surface flow is considered. The structure deformations are assumed to be very small, so that the interface Σ is fixed. The governing Navier-Stokes equation and the boundary conditions for this problem are:

$$\rho_f \frac{\partial \mathbf{u}}{\partial t} + \rho_f \mathbf{u}.\nabla \mathbf{u} = -\nabla p + \mu \Delta \mathbf{u} + \rho_f \mathbf{g} \quad \text{in} \quad \Omega_f = \Omega_f(t) \quad (1)$$

$$\nabla.\mathbf{u} = 0 \quad \text{in} \quad \Omega_f = \Omega_f(t) \quad (2)$$

$$\mathbf{u}.\mathbf{n} = 0 \quad \text{on} \quad \Gamma_f = \Gamma_f(t) \quad (3)$$

$$\mathbf{u}.\mathbf{n} = \mathbf{u}|_{fs}.\mathbf{n} \quad \text{on} \quad \Gamma_{fs} = \Gamma_{fs}(t) \quad (4)$$

where $\mathbf{u}$ is the velocity vector, p the pressure, ρ_f fluid density, μ fluid viscosity and $\mathbf{g}$ is the vector of gravity.
The free surface is tracked using the VOF method, the volume fractions ϕ are advected by the velocity field [6]. The advection is governed by the transport equation, which, for an incompressible flow, reads:

$$\frac{\partial \phi}{\partial t} + \nabla.(\mathbf{u}\phi) = 0 \tag{5}$$

$$\rho_f = \rho_f(\phi) \tag{6}$$

Structure Domain

For simplicity, the structure is selected to be a one dimensional Euler-Bernoulli beam. For the sake of this study, the structure model itself does not play a rule in the coupling and justification of the new coupling scheme. Assuming constant cross section (A) for the beam, the equation of motion of the beam and it's boundary conditions are:

$$\rho_s A \frac{\partial^2 d}{\partial t^2} + EI \frac{\partial^4 d}{\partial y^4} = f \quad \text{in} \quad \Omega_s = \Omega_s(t) \tag{7}$$

$$d = 0 \quad \text{in} \quad \Omega_s(t=0) \tag{8}$$

$$\frac{\partial d}{\partial t} = 0 \quad \text{on} \quad y = 0 \times (0,T) \tag{9}$$

$$d = 0 \quad \text{on} \quad y = 0 \times (0,T) \tag{10}$$

where d is beam deformation, ρ_s beam density, A beam cross section, E Young modulus, I second moment of inertia and f is the force per length of the beam.

Fluid-Structure Interface

The moment the fluid reaches the beam the wet part of the beam is the interface of the fluid and the structure named Σ. The interface is a subset of the structure domain Ω_s. At the interface, kinematic and dynamic balance should satisfy. reads,

$$\mathbf{u}.\mathbf{n} = \frac{\partial d}{\partial t} \quad \text{on} \quad \Sigma \tag{11}$$

$$f = -\mathrm{h}p \quad \text{on} \quad \Sigma \tag{12}$$

where $\mathbf{n}$ is the outward vector to fluid at the fluid-structure interface and h is the beam width. Eqn.11 is written only for one side of the beam, the side which the fluid outward vector is aligned with structure normal vecor. The interface of the fluid and the structure Γ_f is assumed to be fixed in time.

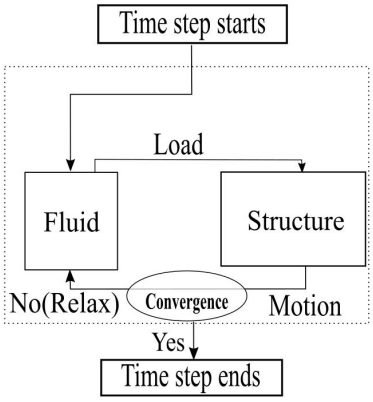

FIGURE 2. THE SCHEMATIC VIEW OF DIRICHLET/NEUMAN ITERATIVE PROCEDURE.

IMPLICIT TIME MARCHING SCHEME

Equations[1-12] are solved by an implicit sequential staggered coupling method. The fluid is solved by a fractional step method based on finite volume and the structure is solved using finite element method. The Dirichlet/Newman coupling algorithm is as follows:

Fluid-Elastic-Structure System

At each time step the iterative coupling algorithm with k as the iteration number indicator as illustrated in Fig.2, is:

(1) Predict structural displacements at the new time level ($\tilde{d}^k$)
(2) Apply relaxation factor ω to the structural displacement for (d^{k+1})
(3) Calculate fluid velocities and pressures at new time level ($\mathbf{u}^{k+1}$, p^{k+1}) using the relaxed structural displacement (d^{k+1})
(4) Update force acting on structure from Eqn.12 (f^{k+1})
(5) Solve the structure response to the loading (f^{k+1}) for ($\tilde{d}^{k+1}$)
(6) Check for convergence of the residual of new structure and fluid velocity at the interface, $\left\| \mathbf{u}^{k+1}.i - \dot{\tilde{d}}^{k+1} \right\|$ which are computed at steps (3) and (5), respectively.
(7) If the convergence is not achieved, predicted deformations can be updated by the computed deformations ($\tilde{d}^{k+1}$) and then go to step (2) until the convergence criteria is satisfied.

Fluid Time Integrator

In this section the fractional step method performed on fluid equations is elaborated. The time discretization of the fluid is performed by means of implicit Euler except for nonlinear con-

vective term and also diffusive term. Discrete form of continuity Eqn.2 is:

$$\mathsf{G}_I^{\mathsf{T}}\mathbf{u}_I^{n+1} = -\mathsf{G}_\Sigma^{\mathsf{T}}\mathbf{u}_\Sigma^{n+1} \tag{13}$$

In this equation, G^{T} is the discrete divergence matrix and indices I and Σ corresponds to interior and fluid-structure interface continuity cells, respectively. The discrete momentum equation is:

$$\rho_f \mathsf{M}^{\mathsf{F}}\dot{\mathbf{u}}_I^{n+1} = -\rho_f \mathsf{C}^{\mathsf{F}}\mathbf{u}_I^n + \rho_f \mathsf{D}^{\mathsf{F}}\mathbf{u}_I^n + \mathsf{G}_I \mathsf{p}_I^{n+1} + \rho_f \mathbf{g}_I \tag{14}$$

In this relation, M^{F} is discrete fluid mass matrix, C^{F} is the discrete convective operator in which is linearized by using velocities at previous time step, D is the discrete diffusive operator and G is discrete gradient operator. By defining the predictor of velocity as follows:

$$\tilde{\mathbf{u}}_I^{n+1} = \mathbf{u}_I^n + \Delta t(\mathsf{M}^{\mathsf{F}})^{-1}(-\mathsf{C}^{\mathsf{F}}\mathbf{u}_I^n + \mathsf{D}^{\mathsf{F}}\mathbf{u}_I^n + \mathbf{g}_I) \tag{15}$$

The Eqn.14 can be rewritten as :

$$\mathbf{u}_I^{n+1} = \tilde{\mathbf{u}}_I^n + \frac{\Delta t}{\rho_f}(\mathsf{M}^{\mathsf{F}})^{-1}\mathsf{G}_I \mathsf{p}_I^{n+1} \tag{16}$$

Taking the divergence of this relation and using the relation in Eqn.13, the pressure Poisson relation is derived,

$$\mathsf{G}_I^{\mathsf{T}}(\mathsf{M}^{\mathsf{F}})^{-1}\mathsf{G}_I \mathsf{p}_I^{n+1} = -\frac{\rho_f}{\Delta t}(\mathsf{G}_I^{\mathsf{T}}\tilde{\mathbf{u}}_I^n + \mathsf{G}_\Sigma^{\mathsf{T}}\mathbf{u}_\Sigma^{n+1}) \tag{17}$$

Structure Solver

The structural response is solved with finite element method, considering Eqn.7 the weak form of the solution using Hermite shape functions ($N_i, i = 1, \cdots, 4$) per element which are $C1$ continuous in space and describe the deformation as $d = N\mathsf{d}$, is:

$$\int_{x_a}^{x_b}(N_i\rho_s A N\ddot{\mathsf{d}} + N_{i,xx}EIN_{xx}\mathsf{d} + N_i f)\mathrm{d}x = \\ +N(x_a)V(x_a) - (-\frac{dN}{dx}\Big|_{x_a})M(x_a) - N(x_b)V(x_b) - (-\frac{dN}{dx}\Big|_{x_b})M(x_b) \tag{18}$$

where test function N_i is selected to be the same as the shape functions. Taking the integrals of the shape functions for each term, will give us the discrete mass operator (M^{S}) and discrete stiffness operator (K^{S}). It reads,

$$\mathsf{M}^{\mathsf{s}}\ddot{\mathsf{d}} + \mathsf{K}^{\mathsf{s}}\mathsf{d} = \mathsf{f} \tag{19}$$

Structure Time Integrator

Temporal discretisation of the structure equations is performed by means of Generalized-α method. With the Generalized-α method, the time levels are applied to the Eqn.19 as:

$$\mathsf{M}^{\mathsf{s}}\ddot{\mathsf{d}}^{\alpha_m} + \mathsf{K}^{\mathsf{s}}\mathsf{d}^{\alpha_f} = \mathsf{f}^{\alpha_f} \tag{20}$$

Acceleration and deformations of the structure are interpolated between discrete time levels by:

$$\ddot{\mathsf{d}}^{\alpha_m} = \alpha_m \ddot{\mathsf{d}}^{n+1} + (1-\alpha_m)\ddot{\mathsf{d}}^n \tag{21}$$
$$\mathsf{d}^{\alpha_f} = \alpha_f \mathsf{d}^{n+1} + (1-\alpha_f)\mathsf{d}^n \tag{22}$$
$$\mathsf{f}^{\alpha_f} = \alpha_f \mathsf{f}^{n+1} + (1-\alpha_f)\mathsf{f}^n \tag{23}$$

The acceleration is related to the displacement and the velocity as:

$$\mathsf{d}^{n+1} = \mathsf{d}^n + \Delta t\dot{\mathsf{d}}^n + \Delta t^2((\frac{1}{2}-\beta)\ddot{\mathsf{d}}^n + \beta\ddot{\mathsf{d}}^{n+1}) \tag{24}$$
$$\dot{\mathsf{d}}^{n+1} = \dot{\mathsf{d}}^n + \Delta t((1-\gamma)\ddot{\mathsf{d}}^n + \gamma\ddot{\mathsf{d}}^{n+1}) \tag{25}$$

In which β and γ are algorithmic factors of the time integrator. In [7] the values of these two factors is related to the spectral radius (ρ_∞). When spectral radios is zero the amount of numerical dissipation is maximum, and when this value is one the dissipation is minimum.

Corrector at The Interface

The fluid velocity and structural velocities should match at the interface of fluid and the structure to satisfy the kinematic coupling relation as Eqn.11. Here the corrector is defined as :

$$\mathbf{u}_\Sigma^{k+1}.\mathbf{n} = \dot{\mathsf{d}}_\Sigma^{k+1} \tag{26}$$

where k indicates the number of iteration in sub-iterative loop.

STABILITY

Observed Instability

The discrete staggered coupled system of Eqn.[11-12, 15 -16, 20-23] are considered. Solving this system exhibit an inherent instability which is not dependent on time step. This problem has been described in [2] and [3]. The is mathematically formulated by Causin et al. [4]. The mass ratio of the fluid to the structure significantly affects the stability. The source of instability of the iterative procedure should be identified by looking at the maximum eigenvalue of the iteration amplification operator.

Added Mass Operator

In order to study the stability of the system, fluid equations are considered in matrix form as Eqn.13 and Eqn.14. In the view of the analysis of the coupled scheme internal fluid forces (convective and diffusive terms) and external forces are neglected. The only forcing which is included is the structure-fluid force along Σ [4].

For the sake of this investigation some assumptions and simplifications are made.

- Both fields equations are regarded as linear
- Since the it is already assumed that the interface of fluid and structure is fixed, the mesh motion due to structure deformations can be neglected as well
- No external forces are assumed except the traction along Σ
- Fluid convective and diffusive terms are omitted from fluid equations
- The cell sizes are assumed to be the same for all the fluid finite volumes

The last assumption is reasonable since for very small time steps the temporal discrete relations are dominant by mass and pressure terms [2]. The assumption of linearity is required for the analysis of maximal eigenvalue of the amplification factor, which is utilized later. Using the assumption that the coefficient matrix G does not change in time, the continuity Eqn.13 can be rewritten as :

$$\mathsf{G}_I^{\mathsf{T}}\dot{\mathbf{u}}_I^{n+1} = -\mathsf{G}_\Sigma^{\mathsf{T}}\dot{\mathbf{u}}_\Sigma^{n+1} \tag{27}$$

If we divide the system into it's coupling interface degrees of freedom on Σ and interior degrees of freedom I,

$$\begin{bmatrix} \mathsf{M}_{\mathsf{II}}^{\mathsf{F}} & 0 & \mathsf{G}_{\mathsf{I}} \\ 0 & \mathsf{M}_{\Sigma\Sigma}^{\mathsf{F}} & \mathsf{G}_\Sigma \\ \mathsf{G}_{\mathsf{I}}^{\mathsf{T}} & \mathsf{G}_\Sigma^{\mathsf{T}} & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}}_{\mathsf{I}} \\ \dot{\mathbf{u}}_\Sigma \\ \mathsf{p} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathsf{f}_\Sigma \\ 0 \end{bmatrix} \tag{28}$$

The coupling force f_Σ can be written in terms of interface acceleration from Eqn.28 as:

$$\mathsf{f}_\Sigma = \Big(\mathsf{G}_\Sigma\big(\mathsf{G}_{\mathsf{I}}^{\mathsf{T}}(\mathsf{M}_{\mathsf{II}}^{\mathsf{F}})^{-1}\mathsf{G}_{\mathsf{I}}\big)^{-1}\big(\mathsf{G}_\Sigma^{\mathsf{T}}\big)^{-1} + \mathsf{M}_{\Sigma\Sigma}^{\mathsf{F}}\Big)\dot{\mathbf{u}}_\Sigma \tag{29}$$

The fluid-structure force at the interface is the acceleration at the interface times the mass. By dividing the mass here with the fluid lumped mass matrix (m^{F}), added mass operator can be derived as:

$$\mathscr{M}_{\mathsf{added}} = \frac{1}{m^{\mathsf{F}}}\Big(\mathsf{G}_\Sigma\big(\mathsf{G}_{\mathsf{I}}^{\mathsf{T}}(\mathsf{M}_{\mathsf{II}}^{\mathsf{F}})^{-1}\mathsf{G}_{\mathsf{I}}\big)^{-1}\big(\mathsf{G}_\Sigma^{\mathsf{T}}\big)^{-1} + \mathsf{M}_{\Sigma\Sigma}^{\mathsf{F}}\Big) \tag{30}$$

The force at the interface then is : $\mathsf{f}_\Sigma = m^{\mathsf{F}}\mathscr{M}_{\mathsf{added}}\dot{\mathbf{u}}_\Sigma$
According to Froster [2], added mass operator contains condensed fluid equations and map the dimensionless acceleration onto dimensionless force. This operator is purely geometrical and it is symmetric and positive. Further simplification step is taken by using lumped fluid mass matrix where the off-diagonal blocks of the mass matrix vanish and the main blocks are replaced by $\mathsf{M}_{\mathsf{II}} = 2m^{\mathsf{F}}\mathsf{I}_{\mathsf{II}}$ and $\mathsf{M}_{\Sigma\Sigma} = m^{\mathsf{F}}\mathsf{I}_{\Sigma\Sigma}$, therefore the added mass operator can be written as:

$$\mathscr{M}_{\mathsf{added}} = \mathsf{I}_{\Sigma\Sigma}^{\mathsf{F}} + 2\mathsf{G}_\Sigma(\mathsf{G}_{\mathsf{I}}^{\mathsf{T}}\mathsf{G}_{\mathsf{I}})^{-1}\mathsf{G}_\Sigma^{\mathsf{T}} \tag{31}$$

In this relation the first part indicates the fluid mass but the second term represents the amount of mass added because of the interaction.

Stability Analysis

The condensed equations of fluid motion in terms of added mass operator as derived in Eqn.31 is employed with structure equations of motion to derive coupled motion, as:

$$\begin{bmatrix} \mathsf{M}_{\mathsf{II}}^{\mathsf{S}} & \mathsf{M}_{\mathsf{I}\Sigma}^{\mathsf{S}} \\ \mathsf{M}_{\Sigma\mathsf{I}}^{\mathsf{S}} & \mathsf{M}_{\Sigma\Sigma}^{\mathsf{S}} \end{bmatrix} \begin{bmatrix} \ddot{\mathsf{d}}_{\mathsf{I}} \\ \ddot{\mathsf{d}}_\Sigma \end{bmatrix} + \begin{bmatrix} \mathsf{K}_{\mathsf{II}}^{\mathsf{S}} & \mathsf{K}_{\mathsf{I}\Sigma}^{\mathsf{S}} \\ \mathsf{K}_{\Sigma\mathsf{I}}^{\mathsf{S}} & \mathsf{K}_{\Sigma\Sigma}^{\mathsf{S}} \end{bmatrix} \begin{bmatrix} \mathsf{d}_{\mathsf{I}} \\ \mathsf{d}_\Sigma \end{bmatrix} = \begin{bmatrix} 0 \\ -\mathsf{f}_\Sigma \end{bmatrix} \tag{32}$$

where K^{S} is structural stiffness matrix obtained from Eqn.19. The stiffness term in Eqn.32 is time step dependent

and for enough small time steps the mass term will be dominate. Since the focus is on non time step dependent stability source, this is term is neglected. Since the structure is a one dimensional beam, the interior elements are actually exactly the interface elements. Using these assumption, Eqn.32 can be written as:

$$\mathsf{M}^{\mathsf{S}}_{\Sigma\Sigma}\ddot{\mathsf{d}}^{\alpha_m}_{\Sigma}+\mathsf{f}^{\alpha_f}_{\Sigma}=0 \tag{33}$$

The stability or instability of Eqn.32 depends on time integrations used on both fluid and structure side. In this study the time integrator of structure is Generalized-α as Eqn.24 to Eqn.23. The most stability with this time integrator is the case when spectral radius is zero ($\rho_\infty = 0$). According to Chung [7], $\alpha_f = 0$ and $\alpha_m = 2$. In this case numerical dissipation is maximum and therefore stability is less restricted. This assumption is only for the purpose of this analysis. Structural acceleration using Eqn.24 and Eqn.21 and considering that $\alpha_m = 2$, will be:

$$\ddot{\mathsf{d}}^{\alpha_m}_{\Sigma}=\frac{1}{\Delta t^2}(2\tilde{\mathsf{d}}^{n+1,k+1}-5\mathsf{d}^{n}+4\mathsf{d}^{n-1}-\mathsf{d}^{n-2}) \tag{34}$$

In this relation $\tilde{\mathsf{d}}^{k+1}$ is the updated structure acceleration after interation k. When convergence is achieved, $\mathsf{d}^{n+1}=\tilde{\mathsf{d}}^{k+1}$. The fluid force f_Σ combining Eqn.29 and Eqn.31 at time level $\alpha_f = 0$ and employing backward Euler time integrator on fluid side would be :

$$\mathsf{f}^{\alpha_f}_{\Sigma}=\frac{1}{\Delta t^2}m_{\mathsf{F}}\mathscr{M}_{added}(\mathsf{d}^{n+1,k}-2\mathsf{d}^{n}+\mathsf{d}^{n-1}) \tag{35}$$

In Eqn.35, d^k is the structural displacement at previous iteration. In the next step, relaxation is applied on motion of the structure acting on the fluid.

$$\mathsf{d}^{n+1,k+1}=\omega\tilde{\mathsf{d}}^{n+1,k+1}+(1-\omega)\mathsf{d}^{n+1,k} \tag{36}$$

After lumping the structural mass in the Eqn.33 and employing temporal discretization of acceleration and fluid forcing as well as the coupling relaxation, we have:

$$\begin{aligned}&m^{\mathsf{s}}(\frac{2}{\omega}\mathsf{d}^{n+1,k+1}-\frac{2(1-\omega)}{\omega}\mathsf{d}^{n+1,k}-5\mathsf{d}^{n}+4\mathsf{d}^{n-1}-\mathsf{d}^{n-2})\\&+m^{\mathsf{F}}\mathscr{M}_{added}(\mathsf{d}^{n+1,k}-2\mathsf{d}^{n}+\mathsf{d}^{n-1}))=0\end{aligned} \tag{37}$$

Added mass operator is a real positive matrix and therefore all deformations in Eqn.37 can be written in terms of added mass operator eigen vectors, i.e. $\mathsf{d}_\Sigma=\sum_i d_i v_i$. The scalar coefficients have to satisfy :

$$\begin{aligned}&(\frac{2}{\omega}\mathsf{d}_i^{n+1,n+1,k+1}-\frac{2(1-\omega)}{\omega}\mathsf{d}_i^{n+1,k}-5\mathsf{d}_i^{n}+4\mathsf{d}_i^{n-1}-\mathsf{d}_i^{n-2})\\&+\frac{m^{\mathsf{F}}}{m^{\mathsf{S}}}\mu_i(\mathsf{d}_i^{n+1,k}-2\mathsf{d}_i^{n}+\mathsf{d}_i^{n-1}))=0\end{aligned} \tag{38}$$

In this relation μ_i is the maximum eigenvalue of the added mass operator. The fixed point iteration will be:

$$(\frac{2}{\omega})\mathsf{d}_i^{n+1,k+1}+(-\frac{2(1-\omega)}{\omega}+\frac{m^{\mathsf{F}}}{m^{\mathsf{S}}}\mu_i)\mathsf{d}_i^{n+1,k}=f(\mathsf{d}_i^{n},\mathsf{d}_i^{n-1},\mathsf{d}_i^{n-2}) \tag{39}$$

The Dirichlet-Neuman coupling converges to a solution if and only if :

$$0<\omega<\frac{4}{2+\frac{\mathsf{m}^{\mathsf{F}}}{\mathsf{m}^{\mathsf{S}}}\mu_{i,max}} \tag{40}$$

In Eqn.40 as the maximum eigenvalue of added mass operator or the ratio of lumped fluid mass to structure mass or equivalently the ratio of densities increases, lower relaxation is required for a stable convergence.

DEFINITION OF INTERACTION LAW

The idea is to incorporate structural velocities into continuity equation of the fluid. Structural velocities are a function of load acting on it. This incorporates pressures into continuity equation fro the fluid. It reads,

$$\dot{\mathsf{u}}_\Sigma\rightarrow\dot{\mathsf{u}}_\Sigma+\mathsf{B}\mathsf{p} \tag{41}$$

where B is the discrete operator which translates pressures into accelerations. An appropriate operator with certain properties will be determined in this section. Any relation between f_Σ and $\mathbf{u}_\Sigma$ which is derived form this system can represent the added mass operator and would be identical to the operator derived in Eqn.32. In order to do so, from Eqn.34 it can be written :

$$\mathsf{M}^{\mathsf{S}}\ddot{\mathsf{d}}^{\alpha_m}+\mathsf{K}^{\mathsf{S}}\mathsf{d}^{\alpha_f}=-\mathsf{f}^{\alpha_f}_{\Sigma} \tag{42}$$

By the means of modal analysis the Eqn.42 can be written in terms of the structure matrix of eigen vectors Q and participation factor of the modes as z:

$$\mathsf{Q}^T\mathsf{M}^{\mathsf{S}}\mathsf{Q}\ddot{\mathsf{z}}^{\alpha_m}+\mathsf{Q}^T\mathsf{K}^{\mathsf{S}}\mathsf{Q}\mathsf{z}^{\alpha_f}=-\mathsf{Q}^T\mathsf{f}_{\Sigma}^{\alpha_f} \tag{43}$$

The eigen vectors are orthogonal and therefore Eqn.43 is decoupled system of equations. This is because the off diagonal entries of matrix coefficients will be zero. We have n decoupled equations in which n is the total discrete degrees of freedom of the structure. Moreover the diagonal entries of mass matrix, can be one with an appropriate scaling on the eigen vectors. So the Eqn.43 is rewritten like :

$$\mathsf{I}\ddot{\mathsf{z}}^{\alpha_m}+\Lambda\mathsf{z}^{\alpha_f}=-\mathsf{Q}^T\mathsf{f}_{\Sigma}^{\alpha_f} \tag{44}$$

Where Λ is a matrix with eigenvalues of the structure at diagonal entries.

Time Integrator of Interaction Law

In Eqn.44, acceleration and displacement participation factors appeared and our aim is to drive a relation between forces and acceleration at the interface. Time integrator can relate structural displacements at new time level to structural acceleration. The role of IL is to mimic the structural response. For this reason, the time integrator of interaction law is selected to be exactly the same as time integrator of the structural solver i.e. Generalized-α. Substituting Eqn.24 into Eqn.44, we will have:

$$(\alpha_m\mathsf{I}+\alpha_f\Delta t^2\beta\Lambda)\ddot{\mathsf{z}}^{n+1}+f(\ddot{\mathsf{z}}^n,\dot{\mathsf{z}}^n,\mathsf{z}^n)=-\alpha_f\mathsf{Q}^T\mathsf{f}_{\Sigma}^{n+1} \tag{45}$$

The acceleration participation factor can be written in terms of interface force, as:

$$\ddot{\mathsf{z}}^{n+1}=-\alpha_f(\alpha_m\mathsf{I}+\alpha_f\Delta t^2\beta\Lambda)^{-1}\mathsf{Q}^T\mathsf{f}_{\Sigma}^{n+1} \tag{46}$$

The accelration at the interface is $\mathbf{u}_{\Sigma}^{n+1}=\ddot{\mathsf{d}}_{\Sigma}^{n+1}=\mathsf{Q}\ddot{\mathsf{z}}^{n+1}$, so we will have:

$$\dot{\mathsf{u}}_{\Sigma}^{n+1}=-\alpha_f\mathsf{Q}(\alpha_m\mathsf{I}+\alpha_f\Delta t^2\beta\Lambda)^{-1}\mathsf{Q}^{\mathsf{T}}\mathsf{f}_{\Sigma}^{n+1} \tag{47}$$

$$\mathsf{f}_{\Sigma}^{n+1}=-\alpha_f\mathsf{Q}^{\mathsf{T}}(\alpha_m\mathsf{I}+\alpha_f\Delta t^2\beta\Lambda)\mathsf{Q}\dot{\mathsf{u}}_{\Sigma}^{n+1} \tag{48}$$

The added mass operator is identified from Eqn.48 and can be written as Eqn.31, like:

$$\mathscr{M}_{\mathsf{added}}=\alpha_f\mathsf{Q}^{\mathsf{T}}(\alpha_m\mathsf{I}+\alpha_f\Delta t^2\beta\Lambda)\mathsf{Q} \tag{49}$$

Eqn. 31 and Eqn.49 are the same since the they have been derived from the relations of a system, but only one by looking at the fluid and the other one from the structural side. In the way the later is presented some interesting properties of this matrix is revealed. As shown previously, when the stability analysis is performed, the maximum eigenvalue of this operator dominates and restricts the stability of the system. This maximum eigenvalue is related to the maximum eigenvalue of the structure.
If this forces are added to the force acting on fluid and subtracted from the structure, basically the system of fluid and structure will be solved monotonically at the structure side.

$$\begin{aligned}\begin{bmatrix}\mathsf{M}_{\mathsf{II}}^{\mathsf{S}} & \mathsf{M}_{\mathsf{I}\Sigma}^{\mathsf{S}}\\ \mathsf{M}_{\Sigma\mathsf{I}}^{\mathsf{S}} & \mathsf{M}_{\Sigma\Sigma}^{\mathsf{S}}\end{bmatrix}\begin{bmatrix}\ddot{\mathsf{d}}_{\mathsf{I}}\\ \ddot{\mathsf{d}}_{\Sigma}\end{bmatrix}+\begin{bmatrix}\mathsf{K}_{\mathsf{II}}^{\mathsf{S}} & \mathsf{K}_{\mathsf{I}\Sigma}^{\mathsf{S}}\\ \mathsf{K}_{\Sigma\mathsf{I}}^{\mathsf{S}} & \mathsf{K}_{\Sigma\Sigma}^{\mathsf{S}}\end{bmatrix}\begin{bmatrix}\dot{\mathsf{d}}_{\mathsf{I}}\\ \dot{\mathsf{d}}_{\Sigma}\end{bmatrix}\\ =\begin{bmatrix}0\\ -(\mathsf{f}_{\Sigma}-\mathsf{Q}^{\mathsf{T}}(\mathsf{I}+\Delta t^2\beta\Lambda)\mathsf{Q}\dot{\mathsf{u}}_{\Sigma})\end{bmatrix}\end{aligned} \tag{50}$$

$$\begin{bmatrix}\mathsf{M}_{\mathsf{II}}^{\mathsf{F}} & 0 & \mathsf{G}_{\mathsf{I}}\\ 0 & \mathsf{M}_{\Sigma\Sigma}^{\mathsf{F}} & \mathsf{G}_{\Sigma}\\ \mathsf{G}_{\mathsf{I}}^{\mathsf{T}} & \mathsf{G}_{\Sigma}^{\mathsf{T}} & 0\end{bmatrix}\begin{bmatrix}\dot{\mathbf{u}}_{\mathsf{I}}\\ \dot{\mathbf{u}}_{\Sigma}\\ \mathsf{p}\end{bmatrix}=\begin{bmatrix}0\\ \mathsf{f}_{\Sigma}-\mathsf{Q}^{\mathsf{T}}(\mathsf{I}+\Delta t^2\beta\Lambda)\mathsf{Q}\dot{\mathsf{u}}_{\Sigma}\\ 0\end{bmatrix} \tag{51}$$

The system of equations as in Eqn.51 can be rewritten as :

$$\begin{bmatrix}\mathsf{M}_{\mathsf{II}}^{\mathsf{F}} & 0 & \mathsf{G}_{\mathsf{I}}\\ 0 & \mathsf{M}_{\Sigma\Sigma}^{\mathsf{F}} & \mathsf{G}_{\Sigma}+\mathsf{Q}^{\mathsf{T}}(\mathsf{I}+\Delta t^2\beta\Lambda)\mathsf{Q}\\ \mathsf{G}_{\mathsf{I}}^{\mathsf{T}} & \mathsf{G}_{\Sigma}^{\mathsf{T}} & 0\end{bmatrix}\begin{bmatrix}\dot{\mathbf{u}}_{\mathsf{I}}\\ \dot{\mathbf{u}}_{\Sigma}\\ \mathsf{p}\end{bmatrix}=\begin{bmatrix}0\\ \mathsf{f}_{\Sigma}\\ 0\end{bmatrix} \tag{52}$$

If the stability analysis of system of equations as in Eqn.52 is performed the added mass operator will completely vanish and therefore the stability criteria for this system will be :

$$0<\omega<2 \tag{53}$$

In which, even for no relaxation $\omega=1$ still will be stable. In practice solving for all structural eigenvalues and values is by itself for very large discrete systems not practical since it can be expensive and for higher modes depending on the numerical method used even inaccurate as the round off errors become

TABLE 1. GENERAL SPECIFICATIONS OF THE BEAM.

Parameter	Value
Length	0.20 m
Thickness	0.02 m
Density	800 kg/m^3
Young modulus	3.4×10^9 Pa

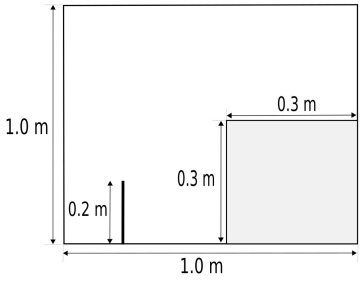

FIGURE 3. THE NUMERICAL SETUP DIMENSIONS

important. A better approach is to add structural mode shapes to the interaction law one by one. In this way, only the mode added to IL will vanish from added mass operator, therefore the next highest value of the added mas operator will be dominant. Since the second highest eigenvalue is always smaller than the first one, stability restriction will be less. This procedure can continue until the number of sub-iterations reduce sufficiently. For very large discrete systems sometime adding very high modes, doesn't improve the approximation of the structure. This is due to the fact the first each mode has na amplification factor which is defining the importance of that mode shape. If the amplification factor is very small, the effect of that mode on the response is not significant. Secondly, since the eigen modes in the IL are computed numerically, they contain numerical error specially for higher modes.

NUMERICAL TEST CASE

The two dimensional dam break problem which the waves interact with an elastic mono-pile with rectangular cross section is selected as the numerical test case. When first wave hits the beam the added mass of the system starts to increase, therefore the performance of the new quasi-simultaneous method can be evaluated properly. The beam properties are presented in Table 1 and the dimensional properties of the domain is shown in Fig.3.

For fluid finite volume method, a 100×100 grids is used. Total simulation time is $2s$ and the time step is 2×10^{-4}. Fluid

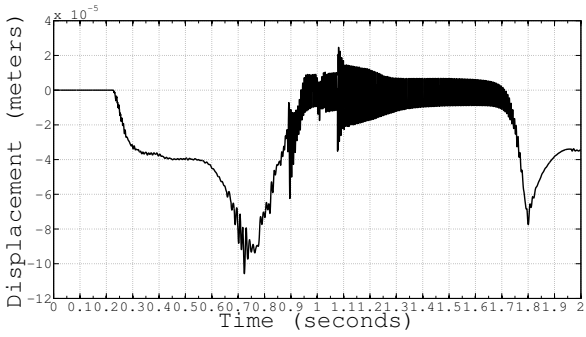

FIGURE 4. DISPLACEMENT OF THE TIP OF THE BEAM

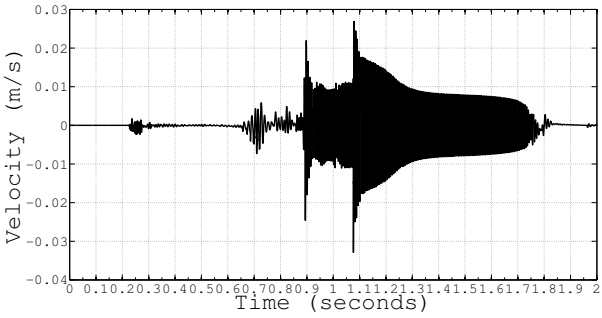

FIGURE 5. VELOCITY OF THE TIP OF THE BEAM

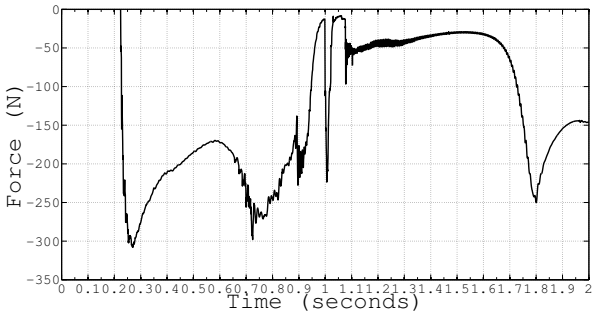

FIGURE 6. TOTAL FORCE ACTING ON THE BEAM

pressure Poisson solver is SOR. For the beam finite element solver, the beam is defined with 20 elements with Hermite shape functions and the spectral radius of the Generalized-α time integrator is 0.7.

Results

After running the simulation, the solution can be presented in terms of the tip of the beam displacement as in Fig.4, the

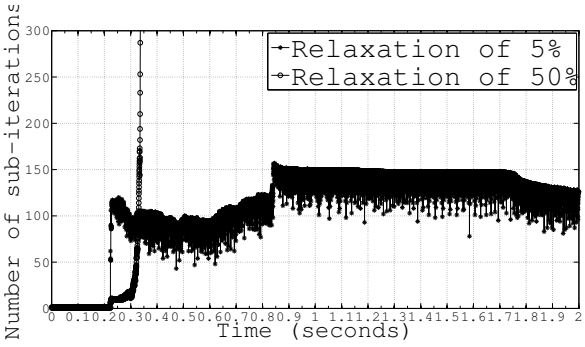

FIGURE 7. TIME HISTORY OF NUMBER OF SUB-ITERATIONS REQUIRED TO ACHIEVE CONVERGENCE PER TIME STEP

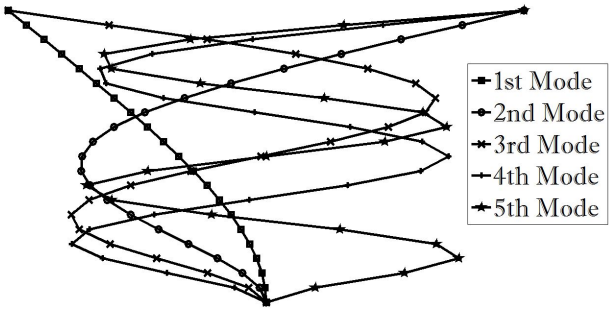

FIGURE 8. FIRST FIVE NUMERICALLY COMPUTED BEAM MODE SHAPES

beam's tip velocity as in Fig.5 and also the total horizontal force acting on beam in Fig.6. As illustrated in displacement and force history, this problem undergoes several stages. initially the fluid is at rest and then it starts to flow. The moment that water hits the beam for the first time, the force acting on the beam starts to increase. The force reaches it's maximum and the water continues going upward while the force lowers. Some part of the water form the reflective wave and the rest end up on the other side of the beam. The next peak in the force is when the reflected wave again hits the beam. Different snapshots of the fluid flow is shown in Fig.10.

As shown in section of stability, this coupling requires an specific amount of under-relaxation to keep it stable. For this purpose two different relaxation values, 0.05 and 0.50 are selected. In Fig.7, it is shown that via 0.50 as the relaxation, the moment the water hits the beam and the added mass increases, coupled scheme tends to diverge. However during intial phase with a very low added mass ratio, the number of sub-iterations are even less than relaxation of 0.05. This is an approve for Eqn.40 when the maximum eigenvalue of added mass operator dominates the limit of stability.

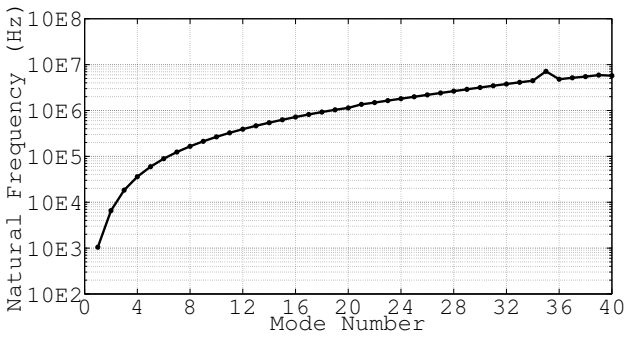

FIGURE 9. ALL NUMERICALLY COMPUTED DISCRETE BEAM NATURAL FREQUENCIES

Interaction Law Performance

Now the effect of equipping coupling scheme with IL and investigating it's stabilization ability is intended. As discussed in the interaction law section, the ingredients of this new quasi simultaneous method are structural mode shapes and natural frequencies. As mentioned before, the beam has 20 elements and each node has two degrees of freedom, therefore total discrete degrees of freedom of the structure is 40. Using mass and stiffness matrices of the discrete beam, mode shapes and natural frequencies are computed. The first five structural mode shapes are shown in Fig.8 and it's natural frequencies are shown in Fig.9. IL itself need to be computationally not expensive, for this reason only the first five modes are included into IL. On the other hand, for higher modes numerical round off error can decrease accuracy of them as shown in Fig.9

As discussed in results section, relaxation of 0.50 cannot keep the coupled scheme within stability range. Introducing the IL, this can change. The comparison has been made of time history of sub-iterations for two cases. One with relaxation of 0.05 and without IL, and the second with relaxation factor of 0.50 equipped with the IL. As shown in Fig.11. The IL has eliminated large eigenvalues of the added mass operator and therefore the relaxation of 0.50 is no longer outside of the stability range. As a consequence, no instability is observed and the number of sub-iterations are kept on average around 10. The number of sub-iterations is directly related to the simulation time, assuming that the new boundary conditions don't increase the number of iterations per Poisson solve. This also has been investigated and observed that they remain the same or even decrease in some some instances. On average the speed up of 12 is achieved.

CONCLUSION

The artificial added mass effect of FESI is not an artifact of numerical method used, but this is inherently within the system. The origin of this instability is investigated. Two factors play

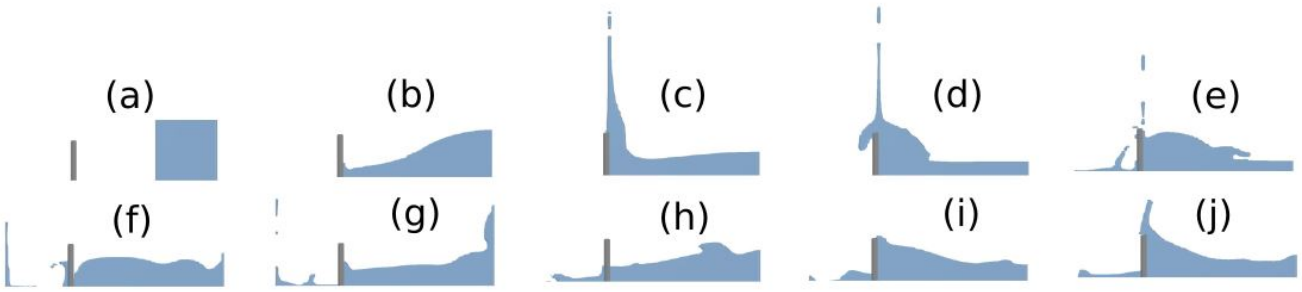

FIGURE 10. TIME HISTORY OF FLUID FLOW, SNAPSHOTS ARE AT TIME 0.00 *s* (a), 0.24 *s* (b), 0.50 *s* (c), 0.72 *s* (d), 0.86 *s* (e), 1.02 *s* (f), 1.24 *s* (g), 1.52 *s* (h), 1.80 *s* (i), 1.98 *s* (j)

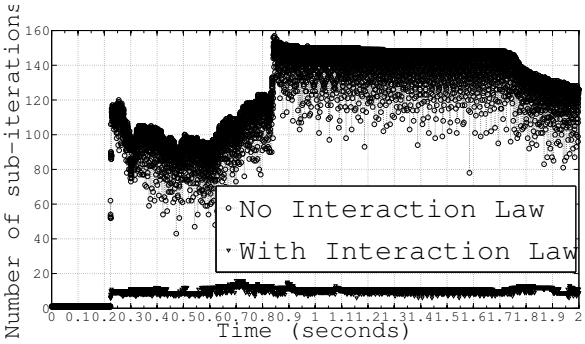

FIGURE 11. COMPARISON BETWEEN NUMBER OF SUB-ITERATIONS AT EACH TIME STEP FOR CONVENTIONAL METHOD WITH NO INTERACTION LAW WITH RELAXATION OF 0.05 AND THE NEW METHOD WITH THE INTERACTION LAW WITH RELAXATION OF 0.50

a role, first the density ratio and second the maximum eigenvalue of the added mass operator. In this study a new method is introduced, which basically treats this problem in a quasi-simultaneous way. The IL is defined based on structural modal representation. This approximate of the structure, essentially removes those highest eigenvalues of the added mass operator depending on number of mode shapes included in IL. The dam break problem interacting with an elastic mono-pile showed that by eliminating the first five highest eigenvalues of added mass operator the relaxation of 0.50 can still be withing the range of stability. While this amount of relaxation do not lead to a stable coupling without the IL. In this case, an overall speed up of 12 was achieved using the new method.

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